

QQQ – Core Pure 2 – Chapter 7 – Methods in Differential Equations (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Find the general solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 2e^x$. [7]

(Total 7 marks)

OCR MEI A2 October 2021 Paper 1 Q13

2. Find the general solution of the differential equation

$$x\frac{dy}{dx} + 5y = \frac{\ln x}{x}, \quad x > 0$$

giving your answer in the form $y = f(x)$. (8)

(Total 8 marks)

Edexcel FP2 June 2011 Q3

3. Given that $3x \sin 2x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4y = k \cos 2x, \quad \text{where } k \text{ is a constant,}$$

(a) calculate the value of k , (4)

(b) find the particular solution of the differential equation for which at $x = 0, y = 2$, and for which at $x = \frac{\pi}{4}, y = \frac{\pi}{2}$. (4)

(Total 8 marks)

Edexcel FP2 June 2006 Q1

Solutions

1.

Scheme	Marks	AO
$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 2e^x$ AE: $\lambda^2 + 2\lambda - 3 = 0 \Rightarrow \lambda = -3, 1$	M1	2.1
CF: $y = Ae^{-3x} + Be^x$	A1	2.1
PI: $y = Cxe^x$	M1	2.1
$\frac{dy}{dx} = C(e^x + xe^x)$	A1	1.1
$\frac{d^2y}{dx^2} = C(2e^x + xe^x)$	A1	1.1
$\Rightarrow C(2e^x + xe^x) + 2C(e^x + xe^x) - 3Cxe^x = 2e^x$ $\Rightarrow 4C = 2 \Rightarrow C = \frac{1}{2}$	M1	2.1
GS: $y = Ae^{-3x} + Be^x + \frac{1}{2}xe^x$	A1	2.2a
	[7]	

2.

Scheme	Marks
$\frac{dy}{dx} + 5\frac{y}{x} = \frac{\ln x}{x^2}$ Integrating factor $e^{\int \frac{5}{x}}$	M1
$e^{\int \frac{5}{x}} = e^{5 \ln x} = x^5$	A1
$\int x^3 \ln x \, dx = \frac{x^4 \ln x}{4} - \int \frac{x^3}{4} \, dx$	M1 M1 A1
$= \frac{x^4 \ln x}{4} - \frac{x^4}{16} (+C)$	A1
$x^5 y = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C \quad y = \frac{\ln x}{4x} - \frac{1}{16x} + \frac{C}{x^5}$	M1 A1
	(8 marks)

1st M1 for attempt at correct Integrating Factor

1st A1 for simplified IF

2nd M1 for $\frac{\ln x}{x^2}$ times their IF to give their ' $x^3 \ln x$ '

3rd M1 for attempt at correct Integration by Parts

2nd A1 for both terms correct

3rd A1 constant not required

4th M1 $x^5 y = \text{their answer} + C$

3.

Mark scheme (a)

Scheme	Marks
$y' = 3 \sin 2x + 6x \cos 2x$	M1
$y'' = 12 \cos 2x - 12x \sin 2x$	A1
Substituting $12 \cos 2x - 12x \sin 2x + 12x \sin 2x = k \cos 2x$	M1
$k = 12$	A1
	(4)

Notes

(corrected from the printed mark scheme: the substitution line is printed with $12x \sin 2$ for the last $12x \sin 2x$)

Mark scheme (b)

Scheme	Marks
General solution is $y = A \cos 2x + B \sin 2x + 3x \sin 2x$	B1
$(0, 2) \Rightarrow A = 2$	B1
$\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow \frac{\pi}{2} = B + \frac{3\pi}{4} \Rightarrow B = -\frac{\pi}{4}$	M1
$y = 2 \cos 2x - \frac{\pi}{4} \sin 2x + 3x \sin 2x$ Needs $y = \dots$	A1
	(4)

	(8 marks)
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