

QQQ – Core Pure 2 – Chapter 7 – Methods in Differential Equations (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. Solve the differential equation

$$\frac{dy}{dx} - 3y = x$$

to obtain y as a function of x .

(5)

(Total 5 marks)

Edexcel FP2 June 2008 Q1

2. Find, in the form $y = f(x)$, the general solution of the differential equation

$$\tan x \frac{dy}{dx} + y = 3 \cos 2x \tan x, \quad 0 < x < \frac{\pi}{2}$$

(6)

(Total 6 marks)

Edexcel FP2 June 2015 Q3

3.

(a) Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$. **[2]**

(b) Hence find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = x(4 - 5x). \quad \mathbf{[4]}$$

(Total 6 marks)

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4.

(a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{3x}$$

giving your answer in the form $y = f(x)$ (5)

Given that $y = 5$ and $\frac{dy}{dx} = 12$ when $x = 0$

(b) determine the particular solution of the differential equation. (3)

(Total 8 marks)

Solutions

1.

Scheme	Marks
Integrating factor = e^{-3x}	B1
$\therefore \frac{d}{dx}(ye^{-3x}) = xe^{-3x}$	M1
$\therefore (ye^{-3x}) = \int xe^{-3x} dx = -\frac{x}{3}e^{-3x} + \int \frac{1}{3}e^{-3x} dx$	M1
$= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x} (+c)$	A1
$\therefore y = -\frac{x}{3} - \frac{1}{9} + ce^{3x}$	A1ft
	(5)
	(5 marks)

Notes

First M for multiplying through by Integrating Factor and evidence of calculus

Second M for integrating by parts 'the right way around'.

Be generous – ignore wrong signs and wrong constants.

Second M dependent on first. Both As dependent on this M.

First A1 for correct expression – constant not required

Second A requires constant for follow through.

If treated as a second order de with errors then send to review.

2.

Scheme	Marks
$\frac{dy}{dx} + \frac{y}{\tan x} = 3 \cos 2x$	
$\int \cot x dx = \ln \sin x $, IF = $\sin x$	M1
$\sin x \frac{dy}{dx} + y \cos x = 3 \cos 2x \sin x$	

$y \sin x = \int 3 \cos 2x \sin x \, dx$	M1A1
$y \sin x = \int 3 (2 \cos^2 x - 1) \sin x \, dx$ or $y \sin x = \frac{3}{2} \int (\sin 3x - \sin x) \, dx$	
$y \sin x = 3 \left[-\frac{2}{3} \cos^3 x + \cos x \right] (+c)$ or $y \sin x = \frac{3}{2} \left[-\frac{1}{3} \cos 3x + \cos x \right] (+c)$	dM1A1
$y = \frac{3 \cos x - 2 \cos^3 x + c'}{\sin x}$ oe or $y = \frac{-3 \cos 3x + 3 \cos x + c'}{2 \sin x}$	Blft (A1 on e-PEN)
	(6 marks)

Notes

M1: Divide by $\tan$ and attempt IF $e^{\int \cot x \, dx}$ or equivalent needed

M1: Multiply through by IF and integrate LHS

A1: correct so far

dM1: dep (on previous M mark) integrate RHS using double angle or factor formula

$$k \cos^2 x \sin x \rightarrow \pm \cos^3 x, \quad k \sin^2 x \cos x \rightarrow k \sin^3 x, \quad \cos 3x \rightarrow \pm \frac{1}{3} \sin 3x, \quad \sin 3x \rightarrow \pm \frac{1}{3} \cos 3x$$

A1: All correct so far constant not needed

Blft: obtain answer in form $y = \dots$ any equivalent form Constant must be included and dealt with correctly. Award if correctly obtained from the previous line

Alternatives for integrating the RHS:

(i) By parts: Needs 2 applications of parts or one application followed by a trig method. Give M1 only if method is complete and A1 for a correct result.

$$(ii) y \sin x = \int 3 (1 - 2 \sin^2 x) \sin x \, dx = \int 3 \sin x - 6 \sin^3 x \, dx$$

Then use $\sin 3x = 3 \sin x - 4 \sin^3 x$ to get $y \sin x = \int \frac{3}{2} (\sin 3x - \sin x) \, dx$ and integration shown above - both steps needed for M1

ALTERNATIVE: Mult through by $\cos x$

Scheme	Marks
$\sin x \frac{dy}{dx} + y \cos x = 3 \cos 2x \sin x$	M1

$y \sin x = \int 3 \cos 2x \sin x \, dx$	M1A1
Rest as main scheme	

3.

Mark scheme (a)

Scheme	Marks
Auxiliary equation: $n^2 - 2n + 5 = 0$ $\Rightarrow n = 1 \pm 2i$	B1
$\Rightarrow y = e^x (A \cos 2x + B \sin 2x)$ oe	B1
	[2]

Notes

B1: Correct roots of auxiliary equation.

B1: ft complex k only.

Or $y = Re^x \cos(2x + \varphi)$ or $y = Re^x \sin(2x + \varphi)$

Or $y = Ae^{(2i+1)x} + Be^{(-2i+1)x}$

Final equation must be $y = f(x)$

Mark scheme (b)

Scheme	Marks
Trial function: $y = ax^2 + bx + c$	B1
$\Rightarrow y' = 2ax + b, \quad y'' = 2a$ $\Rightarrow 2a - 2(2ax + b) + 5(ax^2 + bx + c) \equiv 4x - 5x^2$ $\Rightarrow 5ax^2 + x(-4a + 5b) + 5c - 2b + 2a \equiv 4x - 5x^2$	M1
$\Rightarrow a = -1, b = 0, c = \frac{2}{5}$	A1
$\Rightarrow$ GS: $y = e^x (A \cos 2x + B \sin 2x) - x^2 + \frac{2}{5}$	A1ft
	[4]

Notes

B1: Allow any extraneous terms in the trial function (eg. dx^3) as long as d shown to be zero, $a, b, c \neq 0$

M1: Differentiates their trial function to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and substitutes

A1ft: **ft** their particular integral, and their CF from (a) (dependent on CF containing exactly two arbitrary constants)

4.

Mark scheme (a)

Scheme	Marks	AO
$m^2 - 4m + 4 = 0 \Rightarrow m = \dots \{2\}$	M1	1.1b
$(y) = Ae^{2x} + Bxe^{2x}$	A1	1.1b
PI $y = \lambda e^{3x}$, $\frac{dy}{dx} = 3\lambda e^{3x}$ and $\frac{d^2y}{dx^2} = 9\lambda e^{3x}$ $9\lambda e^{3x} - 4(3\lambda e^{3x}) + 4(\lambda e^{3x}) = 2e^{3x}$ leading to $\lambda = \dots$	M1	3.1a
$(y) = CF + PI$	dM1	1.1b
$y = Ae^{2x} + Bxe^{2x} + 2e^{3x}$	A1	1.1b
	(5)	

Notes

M1: Forms the correct auxiliary equation, leading to finding a value for m .

Allow the use of any variable instead of m for this mark.

A1: Correct complementary function, oe such as $y = (A + Bx)e^{2x}$

Do not need $y =$ here but **must** be in terms of x , but may be recovered later.

M1: A complete method to find the particular integral. Uses the correct form $y = \lambda e^{3x}$, differentiates twice and substitutes correctly into the differential equation to find a value for λ ($= 2$). May use a different variable for x here.

dM1: Dependent on the previous method mark. Finds the general solution by adding the particular integral (of the correct form) to their complementary function (which may not be of the correct form). Do not need $y =$ here. May use a different variable for x but their variable must be consistent.

A1: Correct general solution. **Must** have $y =$ here and **must** be in terms of x

Mark scheme (b)

Scheme	Marks	AO
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$x = 0, y = 5 \Rightarrow 5 = A + 2 \Rightarrow A = \dots$	M1	1.1b
$\frac{dy}{dx} = 2Ae^{2x} + Be^{2x} + 2Bxe^{2x} + 6e^{3x}$ and uses their value of A and $x = 0, \frac{dy}{dx} = 12$ to find the value of B $12 = 6 + B + 6 \Rightarrow B = \dots$	dM1	3.1a
$y = 3e^{2x} + 2e^{3x}$	A1	1.1b
	(3)	
	(8 marks)	

Notes

M1: For a general solution of the form $Ae^{\alpha x} + Bxe^{\alpha x} + \text{“their PI”}$, uses the initial conditions, $x = 0, y = 5$ to find a value for a constant.

(The correct form will be $Ae^{mx} + Bxe^{mx} + \lambda e^{3x}$)

dM1: Dependent on the previous method mark.

Differentiates the general solution to achieve an expression of the form

$mAe^{mx} + Be^{mx} + mBxe^{mx} + 3\lambda e^{3x}$ using their m from part (a) where $m \neq 1$

Then uses the initial conditions to find a value for the other constant.

A1: Correct particular solution. Must have $y =$ here and must be in terms of x