

QQQ – Core Pure 2 – Chapter 4 – Volumes of Revolution (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1.

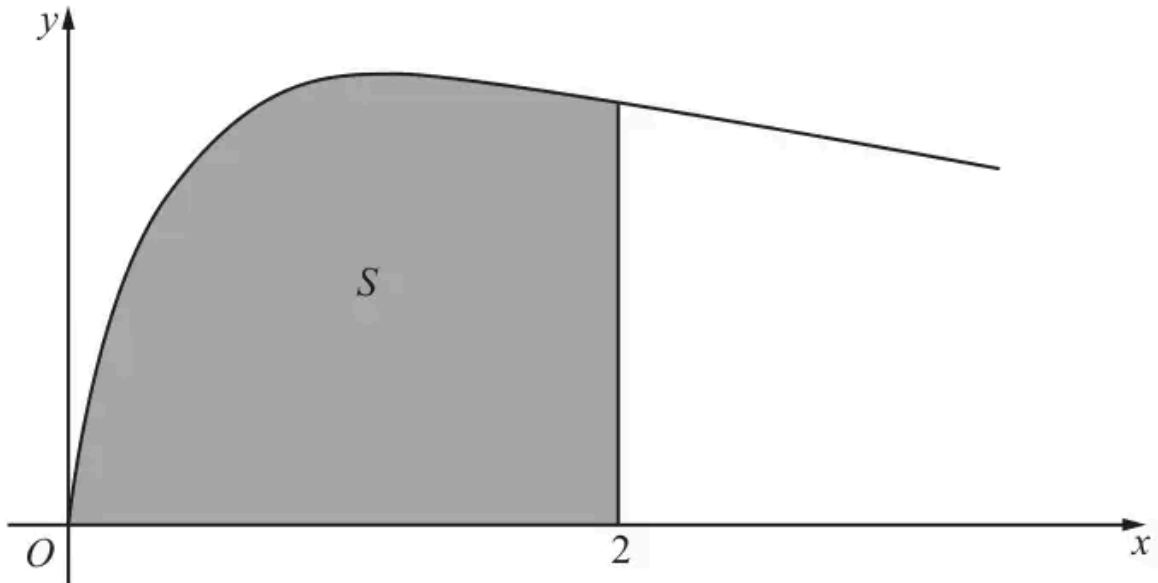


Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geq 0$$

The finite region S , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 2$

The region S is rotated 360° about the x -axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants. (5)

(Total 5 marks)

2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

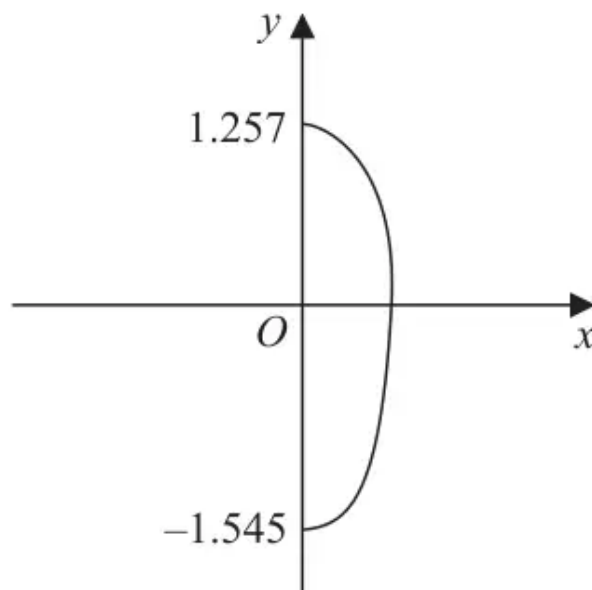


Figure 2

John picked 100 berries from a plant.

The largest berry picked was approximately 2.8 cm long.

The shape of this berry is modelled by rotating the curve with equation

$$16x^2 + 3y^2 - y \cos\left(\frac{5}{2}y\right) = 6 \quad x \geq 0$$

shown in Figure 2, about the y -axis through 2π radians, where the units are cm.

Given that the y intercepts of the curve are -1.545 and 1.257 to four significant figures,

(a) use algebraic integration to determine, according to the model, the volume of this berry.

(6)

Given that the 100 berries John picked were then squeezed for juice,

(b) use your answer to part (a) to decide whether, in reality, there is likely to be enough juice to fill a 200 cm^3 cup, giving a reason for your answer.

(2)

(Total 8 marks)

3.

- (a) Using the identity $\cos 2\theta = 1 - 2\sin^2 \theta$, find $\int \sin^2 \theta \, d\theta$. (2)

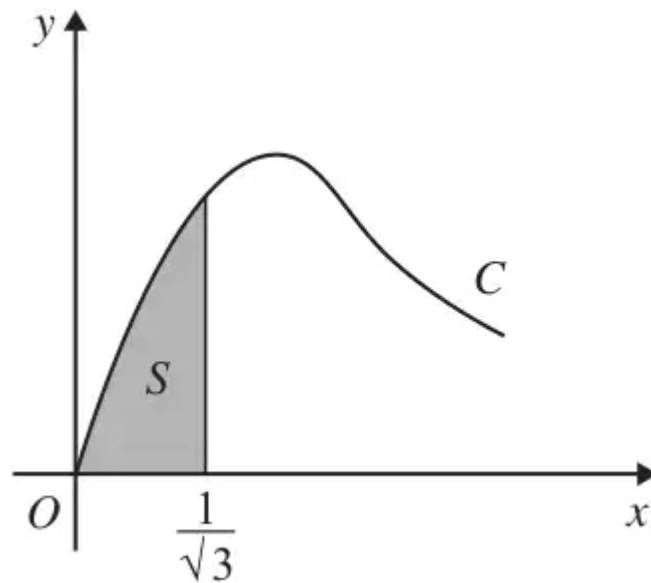


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan \theta, \quad y = 2 \sin 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The finite shaded region S shown in Figure 4 is bounded by C , the line $x = \frac{1}{\sqrt{3}}$ and the x -axis.

This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Show that the volume of the solid of revolution formed is given by the integral

$$k \int_0^{\frac{\pi}{6}} \sin^2 \theta \, d\theta$$

where k is a constant. (5)

- (c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants. (3)

(Total 10 marks)