

QQQ – Core Pure 2 – Chapter 4 – Volumes of Revolution (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. In this question you must show detailed reasoning.

Fig. 4 shows the region bounded by the curve $y = \sec \frac{1}{2}x$, the x -axis, the y -axis and the line $x = \frac{1}{2}\pi$.

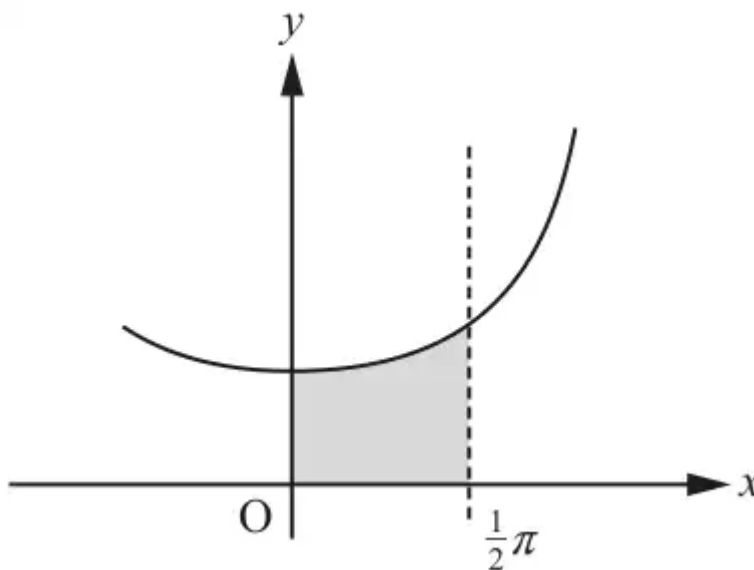


Fig. 4

This region is rotated through 2π radians about the x -axis.

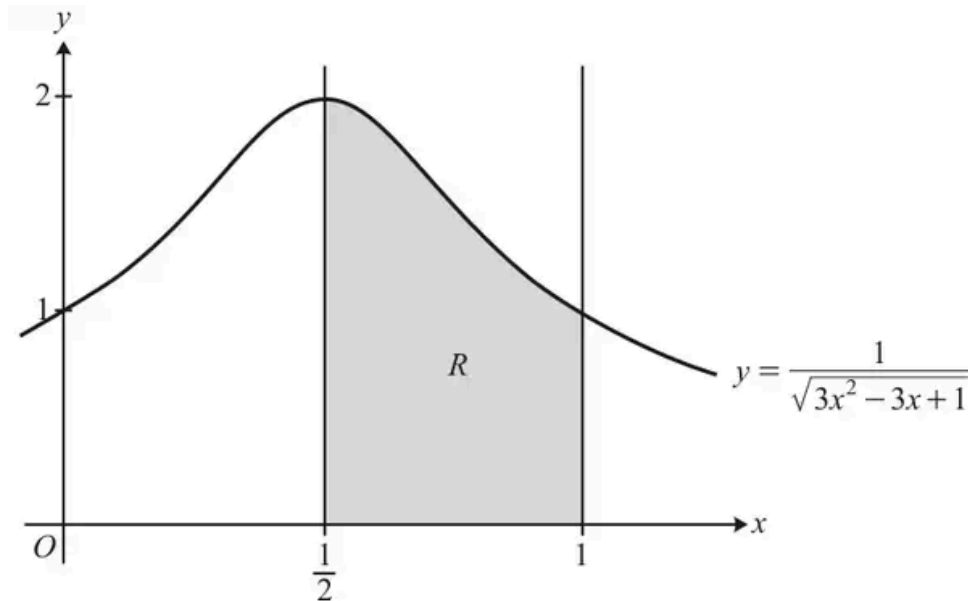
Find, in exact form, the volume of the solid of revolution generated.

[3]

(Total 3 marks)

2. In this question you must show detailed reasoning.

The region R is bounded by the curve with equation $y = \frac{1}{\sqrt{3x^2 - 3x + 1}}$, the x -axis and the lines with equations $x = \frac{1}{2}$ and $x = 1$ (see diagram). The units of the axes are cm.



A pendant is to be made out of a precious metal. The shape of the pendant is modelled as the shape formed when R is rotated by 2π radians about the x -axis.

Find the exact value of the volume of precious metal required to make the pendant, according to the model. **[4]**

(Total 4 marks)

3. The region R_1 is bounded by the curve $y = \frac{80}{\sqrt{25-x^2}}$, the line $x = k$ and the coordinate axes, as shown in **Fig. 1**.

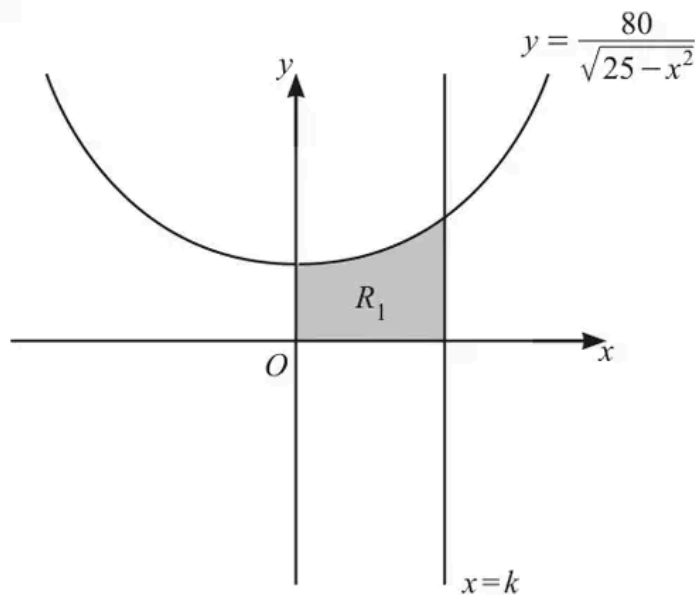


Fig. 1

- (a) In this question you must show detailed reasoning.

Given that the area of R_1 is $\frac{40}{3}\pi$, determine the value of k .

[3]

The region R_2 is bounded by the curve $y = \frac{80}{\sqrt{25-x^2}}$, the line $y = 20$ and the y -axis as shown in **Fig. 2**.

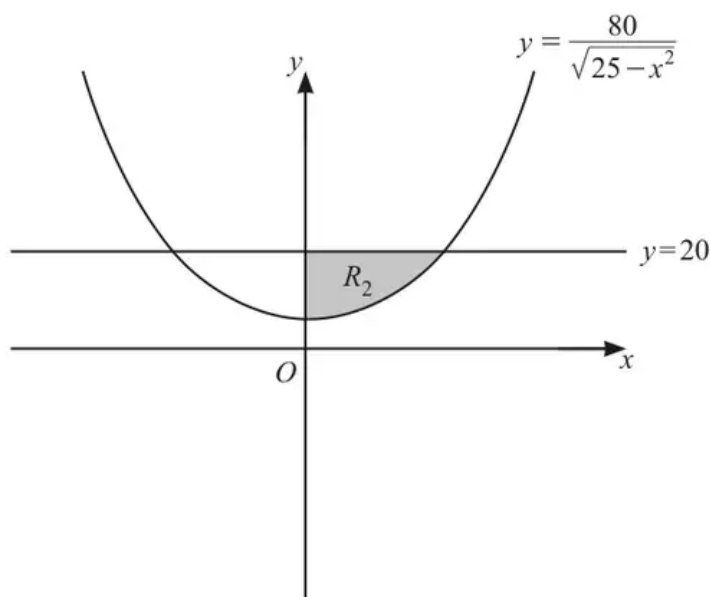


Fig. 2

- (b) Find, in an exact form, the volume of the solid formed when R_2 is rotated by 2π radians about the **y-axis**. **[3]**

(Total 6 marks)

4.

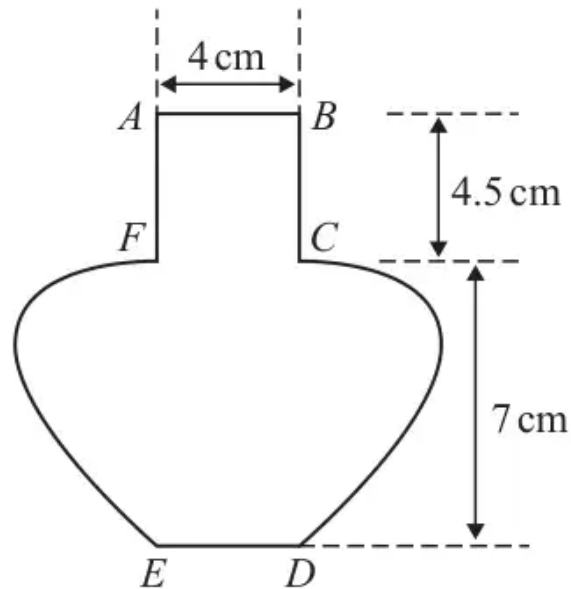


Figure 1

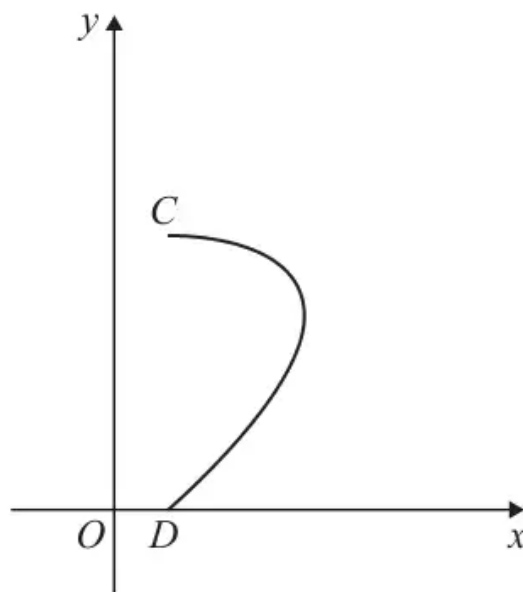


Figure 2

Figure 1 shows the central vertical cross-section $ABCDEF A$ of a vase together with measurements that have been taken from the vase.

The horizontal cross-section between AB and FC is a circle with diameter 4 cm.

The base of the vase ED is horizontal and the point E is vertically below F and the point D is vertically below C .

Using these measurements, the curve CD is modelled by the parametric equations

$$x = a + 3 \sin 2t \quad y = b \cos t \quad 0 \leq t \leq \frac{\pi}{2}$$

where a and b are constants and O is the fixed origin, as shown in Figure 2.

- (a) Determine the value of a and the value of b according to the model. (2)
- (b) Using algebraic integration and showing all your working, determine, according to the model, the volume of the vase, giving your answer to the nearest cm^3 (7)
- (c) State a limitation of the model. (1)

(Total 10 marks)