

QQQ – Core Pure 2 – Chapter 3 – Methods in Calculus (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Find the mean value of $f(x) = x^2 + 6x$ over the interval $[0, 3]$. [2]

(Total 2 marks)

OCR A A2 October 2020 Paper 1 Q1

2. In this question you must show detailed reasoning.

Find the gradient of the curve $y = 6 \arcsin(2x)$ at the point with x -coordinate $\frac{1}{4}$. Express the result in the form $m\sqrt{n}$, where m and n are integers. [4]

(Total 4 marks)

OCR MEI A2 October 2021 Paper 1 Q2

3. In this question you must show detailed reasoning.

Evaluate $\int_0^{\frac{1}{2}} \frac{2}{x^2 - x + 1} dx$. Give your answer in exact form. [4]

(Total 4 marks)

OCR MEI A2 June 2025 Paper 1 Q10

4. In this question you must show detailed reasoning.

Show that $\int_5^{\infty} (x - 1)^{-\frac{3}{2}} dx = 1$. [5]

(Total 5 marks)

OCR A A2 June 2019 Paper 2 Q3

5. In this question you must show detailed reasoning.

By first expressing $\frac{1}{x^2 - 4}$ in partial fractions, show that $\int_3^{\infty} \frac{1}{x^2 - 4} dx = \frac{1}{m} \ln n$, where m and n are integers to be determined. [8]

(Total 8 marks)

OCR MEI A2 June 2025 Paper 1 Q7

Solutions

1.

Scheme	Marks	AO
Mean value = $\frac{1}{3} \int_0^3 f(x) \, dx$	M1	1.1
$= \frac{1}{3} \int_0^3 x^2 + 6x \, dx = 12$	A1	1.1
	[2]	

Notes

M1: Use the correct formula

A1: BC

2.

Scheme	Marks	AO
DR	B1	1.1
$\frac{dy}{dx} = 6 \times \frac{1}{\sqrt{1-4x^2}} \times 2$	M1	1.1
	A1	1.1
when $x = \frac{1}{4}$, $\frac{dy}{dx} = 6 \times \frac{1}{\sqrt{\frac{3}{4}}} \times 2 = 8\sqrt{3}$	A1	1.1
	[4]	

Notes

B1: $\frac{k}{\sqrt{1-4x^2}}$

M1: Chain rule (x 2)

A1: Fully correct $\frac{dy}{dx}$

Alternative solution

Scheme	Marks
DR	M1
$\sin \frac{y}{6} = 2x$	M1

$\frac{dx}{dy} = \frac{1}{12} \cos \frac{y}{6}$	
$\frac{dy}{dx} = \frac{12}{\cos \frac{y}{6}}$ and $x = \frac{1}{4}, y = \pi$	A1
$\rightarrow \frac{dy}{dx} = 8\sqrt{3}$	A1
	[4]

M1: For $\frac{dx}{dy} = k \cos \frac{y}{6}$

M1: Using chain rule ($\times \frac{1}{6}$)

A1: Or $\frac{dy}{dx} = 6 \times \frac{1}{\sqrt{1-4x^2}} \times 2$

(Corrected from the printed mark scheme: the guidance for the first M1 is printed as $\frac{dx}{dy} = k \cos \frac{\pi}{6}$; it should be $k \cos \frac{y}{6}$, as typed above.)

3.

Scheme	Marks	AO
DR $= 2 \int_0^{\frac{1}{2}} \frac{1}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}} dx$	M1*	3.1a
$= \left[\frac{4}{\sqrt{3}} \arctan \left(\frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \right]_0^{\frac{1}{2}} \text{ or } \left[\frac{4}{\sqrt{3}} \arctan \left(\frac{2x-1}{\sqrt{3}} \right) \right]_0^{\frac{1}{2}}$	A1	2.1
$= \frac{4}{\sqrt{3}} \left(\arctan(0) - \arctan \left(\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \right)$	M1dep	2.1
$= \frac{2\pi}{3\sqrt{3}}$	A1cao	1.1
	[4]	

Notes

M1*: completing the square correctly on $x^2 - x + 1$; need not be in integral but must be seen

A1: oe; condone missing or incorrect limits. Could be in terms of another variable if substitution used for

integration, e.g. if $u = x - \frac{1}{2}$ then $\left[\frac{4}{\sqrt{3}} \arctan \left(\frac{u}{\frac{\sqrt{3}}{2}} \right) \right]_{-\frac{1}{2}}^0$ or if $\frac{\sqrt{3}}{2} \tan u = x - \frac{1}{2}$ then $\left[\frac{4}{\sqrt{3}} u \right]_{-\frac{\pi}{6}}^0$.

Condone $\left[\frac{4}{\sqrt{3}} \arctan \left(\frac{x}{\frac{\sqrt{3}}{2}} \right) \right]_{-\frac{1}{2}}^0$ if the correct substitution has been clearly made. Must be seen.

M1dep: using correct limits correctly, including following any substitution. Substitution into (or evaluation of) any non-zero terms must be seen

A1cao: or simplified, exact equivalent. www.

4.

Scheme	Marks	AO
DR $\int (x-1)^{-\frac{3}{2}} dx = -2(x-1)^{-\frac{1}{2}} (+c)$	B1	1.1a
$\int_5^N (x-1)^{-\frac{3}{2}} dx = \left[-2(x-1)^{-\frac{1}{2}} \right]_5^N$	M1	2.1
$-\frac{2}{\sqrt{N-1}} + \frac{2}{\sqrt{5-1}}$	A1	1.1
$\lim_{N \rightarrow \infty} \frac{1}{\sqrt{N-1}} = 0 \text{ oe}$	B1	2.1
$\int_5^\infty (x-1)^{-\frac{3}{2}} dx = \lim_{N \rightarrow \infty} \left\{ -\frac{2}{\sqrt{N-1}} + \frac{2}{\sqrt{5-1}} \right\} = 1$	A1	2.2a
	[5]	

Notes

M1: Consideration of a finite upper limit

B1: Not *just* eg $\frac{1}{\infty} = 0$. Can be seen as part of limit of both terms, but must be explicitly shown as zero

A1: AG. Convincing argument equating improper integral to solution

5.

Scheme	Marks	AO
DR $\frac{1}{x^2 - 4} = \frac{1}{(x-2)(x+2)}$	B1	1.1
$\frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$	M1	1.1
$1 = A(x+2) + B(x-2)$	M1	1.1

$x = 2 \Rightarrow A = \frac{1}{4}, x = -2 \Rightarrow B = -\frac{1}{4}$	A1	1.1
$\int_3^\infty \frac{1}{x^2 - 4} dx = \frac{1}{4} \int_3^\infty \left(\frac{1}{x - 2} - \frac{1}{x + 2} \right) dx$ $= \frac{1}{4} \lim_{k \rightarrow \infty} [\ln(x - 2) - \ln(x + 2)]_3^k$	M1*	2.1
$= \frac{1}{4} \lim_{k \rightarrow \infty} \left[\ln \left(\frac{x - 2}{x + 2} \right) \right]_3^k$	A1	1.1
As $k \rightarrow \infty \frac{k - 2}{k + 2} \rightarrow 1$ or $\ln \left(\frac{k - 2}{k + 2} \right) \rightarrow 0$	A1	2.5
so $\frac{1}{4} \lim_{k \rightarrow \infty} \left[\ln \left(\frac{x - 2}{x + 2} \right) \right]_3^k = -\frac{1}{4} \ln \frac{1}{5} = \frac{1}{4} \ln 5$	B1dep	2.1
	[8]	

Notes

B1: rewriting $\frac{1}{x^2 - 4}$ as $\frac{1}{(x - 2)(x + 2)}$ so

M1: rewriting as partial fractions

M1: evaluating their A and B using substitution, or equating coeffs

A1: $A = \frac{1}{4}$ and $B = -\frac{1}{4}$ or $\frac{1}{4(x - 2)} - \frac{1}{4(x + 2)}$

M1*: integrate their $\left(\frac{1}{x - 2} - \frac{1}{x + 2} \right)$ correctly (condone missing $\frac{1}{4}$ or incorrect multiples). $\frac{1}{4}$ could be incorporated into logarithms.

A1: $\frac{1}{4} [\ln \left(\frac{x - 2}{x + 2} \right)]$. Fraction could be unsimplified.

A1: clear limit argument used to evaluate limit as $k \rightarrow \infty$ or $\lim_{k \rightarrow \infty} \left(\ln \frac{k - 2}{k + 2} \right) = 0$. Must work with a single term.

$\rightarrow$ or $=$ must be used correctly, e.g. do not condone $\lim_{k \rightarrow \infty} \left[\ln \frac{k - 2}{k + 2} \right] \rightarrow 0$ or " $k \rightarrow \infty, \frac{k - 2}{k + 2} = 1$ "

B1dep: Or $m = 4, n = 5$.