

QQQ – Core Pure 2 – Chapter 3 – Methods in Calculus (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Which of the integrals below is **not** an improper integral?

Circle your answer.

[1 mark]

• $\int_0^{\infty} e^{-x} dx$

• $\int_0^2 \frac{1}{1-x^2} dx$

• $\int_0^1 \sqrt{x} dx$

• $\int_0^1 \frac{1}{\sqrt{x}} dx$

(Total 1 marks)

AQA A2 June 2020 Paper 1 Q1

2. The function f is defined by

$$f(x) = \frac{1}{\sqrt{x}} \quad 4 \leq x \leq 7$$

Find the mean value of f over the interval $4 \leq x \leq 7$

Give your answer in exact form.

[3 marks]

(Total 3 marks)

AQA AS June 2024 Paper 1 Q7

3. Given that $y = \sin^{-1}(x^2)$, find $\frac{dy}{dx}$.

[3]

(Total 3 marks)

OCR A A2 June 2024 Paper 1 Q1

4. In this question you must show detailed reasoning.

Find $\int_0^{\frac{1}{3}} \frac{1}{\sqrt{4-9x^2}} dx$, expressing your answer in terms of π .

[4]

(Total 4 marks)

OCR MEI A2 October 2020 Paper 1 Q3

5.

(a) Explain why

$$\int_{\frac{4}{3}}^{\infty} \frac{1}{9x^2 + 16} dx$$

is an improper integral.

(1)

(b) Show that

$$\int_{\frac{4}{3}}^{\infty} \frac{1}{9x^2 + 16} dx = k\pi$$

where k is a constant to be determined.

(4)

(Total 5 marks)

Edexcel A2 June 2024 Paper 2 Q3

6.

(a) Express as partial fractions

$$\frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}$$

(3)

(b) Hence, show that

$$\int_0^2 \frac{2x^2 + 3x + 6}{(x+1)(x^2+4)} dx = \ln(a\sqrt{2}) + b\pi$$

where a and b are constants to be determined.

(4)

(Total 7 marks)

Edexcel A2 June 2022 Paper 1 Q6

Solutions

1.

Scheme	Marks	AO
Circles $\int_0^1 \sqrt{x} \, dx$	B1	2.2a
	(1 mark)	

Typical solution

$$\int_0^1 \sqrt{x} \, dx$$

2.

Scheme	Marks	AO
Writes a correct expression for the mean value, eg $\frac{1}{7-4} \int_4^7 \frac{1}{\sqrt{x}} \, dx$ PI by 0.430...	M1	1.1a
Integrates $\frac{1}{\sqrt{x}}$ to an expression of the form $ax^{\frac{1}{2}}$ where a is non-zero and substitutes 7 and 4 and subtracts. PI by 1.291... or 0.430... Note: $\frac{1}{4} \left(\frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{7}} \right) = 0.433$ is M0	M1	1.1a
Obtains $\frac{2}{3} (\sqrt{7} - 2)$ Ignore an approximated answer.	A1	1.1b
	(3 marks)	

Typical solution

The mean of f

$$\begin{aligned}
&= \frac{1}{7-4} \int_4^7 x^{-\frac{1}{2}} dx \\
&= \frac{1}{3} \left[2x^{\frac{1}{2}} \right]_4^7 \\
&= \frac{2}{3} (\sqrt{7} - 2)
\end{aligned}$$

3.

Scheme	Marks	AO
$\frac{1}{\sqrt{(1 - (x^2)^2)}}$	B1	1.1
$\times 2x$	M1	1.1
$\left(\frac{dy}{dx} = \right) \frac{2x}{\sqrt{1 - x^4}}$	A1	1.1
	[3]	

Notes

B1: For $\frac{1}{\sqrt{(1 - (x^2)^2)}}$ seen.

M1: For $2x \times f(x)$ where $f(x) = \frac{1}{\sqrt{1 - (x^2)^2}}$ or $\frac{1}{\sqrt{1 - x^2}}$ ONLY.

A1: Allow any equivalent correct form e.g. $2x(1 - x^4)^{-0.5}$.

Must be in terms of x .

Condone $(x^2)^2$ for x^4 .

ISW once correct answer seen.

Alternative method

Scheme	Marks
$\cos y \frac{dy}{dx} = 2x$ or $\cos y = 2x \frac{dx}{dy}$	B1
$\sqrt{1 - (x^2)^2} \frac{dy}{dx} = 2x$	M1
$\left(\frac{dy}{dx} = \right) \frac{2x}{\sqrt{1 - x^4}}$	A1
	[3]

B1: For correctly differentiating implicitly with respect to either x or y .

M1: Replacing $\cos y$ with $\pm \sqrt{\pm 1 \pm (x^2)^2}$ in their derivative of the form $\pm \cos y \frac{dy}{dx} = 2x$ (or equivalent if differentiating with respect to y).

A1: Allow any equivalent correct form e.g. $2x(1 - x^4)^{-0.5}$.

Must be in terms of x .

Condone $(x^2)^2$ for x^4 .

ISW once correct answer seen.

4.

Scheme	Marks	AO
DR $\int_0^{\frac{1}{3}} \frac{dx}{\sqrt{4 - 9x^2}} = \frac{1}{3} \int_0^{\frac{1}{3}} \frac{dx}{\sqrt{\frac{4}{9} - x^2}}$	M1	3.1a
$= \left[\frac{1}{3} \arcsin \frac{3x}{2} \right]_0^{\frac{1}{3}}$	A1	1.1
$= \frac{1}{3} \left(\arcsin \frac{1}{2} [- \arcsin 0] \right)$	M1	1.1
$= \frac{\pi}{18}$	A1	1.1
	[4]	

Notes

M1: must be in the form $\frac{k}{\sqrt{\frac{4}{9} - x^2}}, k \neq 1$

A1: $k \arcsin(3x/2)$

can award M1A1 if integral is fully correct before limits substituted

Alternative method

Scheme	Marks
let $x = \frac{2}{3} \sin \theta, \frac{dx}{d\theta} = \frac{2}{3} \cos \theta$	M1
$\int_0^{\frac{1}{3}} \frac{dx}{\sqrt{4 - 9x^2}} = \int_0^{\frac{\pi}{6}} \frac{\frac{2}{3} \cos \theta d\theta}{2 \cos \theta}$	A1
$= \frac{1}{3} [\theta]_0^{\frac{\pi}{6}} = \frac{\pi}{18}$	M1 A1

M1: for suitable substitution

A1: an equivalent expression that can be integrated

M1: substitution of correct limits of their variable

5.

Mark scheme (a)

Scheme	Marks	AO
Because the upper limit is infinite	B1	2.4
	(1)	

Notes

B1: Suitable explanation stating that one of the bounds/limits is infinite or isw
e.g. “one of the limits is unbounded” or “the integral is unbounded”

Do not allow this mark if they only say the limit or integral is undefined, unless they go on to say it is undefined at infinity.

Commenting that the **function** is undefined at infinity is not enough to award this mark on its own, this question concerns the **limits**.

If the candidates state any extra incorrect comments about the **limits** withhold this mark e.g. not defined at $\frac{4}{3}$

Mark scheme (b)

Scheme	Marks	AO
$\int \frac{1}{9x^2 + 16} dx = \frac{1}{12} \arctan\left(\frac{3x}{4}\right)$	M1 A1	3.1a 1.1b
$\int_{\frac{4}{3}}^{\infty} \frac{1}{9x^2 + 16} dx = \frac{1}{12} \lim_{t \rightarrow \infty} \left[\arctan\left(\frac{3x}{4}\right) \right]_{\frac{4}{3}}^t$ $= \frac{1}{12} \left(\lim_{t \rightarrow \infty} \arctan\left(\frac{3t}{4}\right) - \arctan(1) \right)$	dM1	1.1b
$= \frac{1}{12} \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = \frac{\pi}{48}$	A1	2.1
	(4)	
	(5 marks)	

Notes

M1: Integrates to obtain $\alpha \arctan(\beta x)$ where $\beta \neq 1$

A1: Correct integration, unsimplified or simplified.

dM1: Applies correct limits, "t" and $\frac{4}{3}$ with evidence of applying the infinite limit to obtain a non-zero value.

Allow with ∞ used as the limit (which may be implied by $\frac{\pi}{2}$)

A1: Correct value obtained with evidence of use of limiting process on the upper bound. Withhold this mark if there is no evidence of using the limiting process. We must see as a minimum $\lim_{t \rightarrow \infty}$ or at some stage in their work.

e.g. $\left[\frac{1}{12} \arctan \left(\frac{3x}{4} \right) \right]_{\frac{4}{3}}^{\infty} = \frac{1}{12} \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = \frac{\pi}{48}$ would score dM1A0

6.

Mark scheme (a)

Scheme	Marks	AO
$\frac{2x^2 + 3x + 6}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \Rightarrow 2x^2 + 3x + 6$ $= A(x^2+4) + (Bx+C)(x+1)$	M1	1.1b
e.g. $x = -1 \Rightarrow A = \dots, x = 0 \Rightarrow C = \dots, \text{coeff } x^2 \Rightarrow B = \dots$ or Compares coefficients and solves to find values for A, B and C $2 = A + B, 3 = B + C, 6 = 4A + C$	dM1	1.1b
$A = 1, B = 1, C = 2$	A1	1.1b
	(3)	

Notes

M1: Selects the correct form for partial fractions and multiplies through to form suitable identity or uses a method to find at least one value (e.g. cover up rule).

dM1: Full method for finding values for all three constants. Dependent on first M. Allow slips as long as the intention is clear.

A1: Correct constants or partial fractions.

Mark scheme (b)

Scheme	Marks	AO
$\int_0^2 \frac{1}{x+1} + \frac{x+2}{x^2+4} dx = \int_0^2 \frac{1}{x+1} + \frac{x}{x^2+4} + \frac{2}{x^2+4} dx$ $= \left[\alpha \ln(x+1) + \beta \ln(x^2+4) + \lambda \arctan\left(\frac{x}{2}\right) \right]_0^2$	M1	3.1a
$= \left[\ln(x+1) + \frac{1}{2} \ln(x^2+4) + \arctan\left(\frac{x}{2}\right) \right]_0^2$	A1	2.1
$= \left[\ln(3) + \frac{1}{2} \ln(8) + \arctan 1 \right] -$ $\left[\ln(1) + \frac{1}{2} \ln(4) + \arctan(0) \right]$ $= \left[\ln(3) + \frac{1}{2} \ln(8) + \arctan(1) \right] - \left[\frac{1}{2} \ln 4 \right] =$ $\ln\left(\frac{3\sqrt{8}}{2}\right) + \frac{\pi}{4}$	dM1	2.1
$\ln(3\sqrt{2}) + \frac{\pi}{4}$	A1	2.2a
	(4)	
	(7 marks)	

Notes

M1: Splits the integral into an integrable form and integrates at least two terms to the correct form. They may use a substitution on the arctan term

A1: Fully correct Integration.

dM1: Uses the limits of 0 and 2 (or appropriate for a substitution), subtracts the correct way round and combines the ln terms from separate integrals to a single term with evidence of correct ln laws at least once.

A1: Correct answer