

QQQ – Core Pure 2 – Chapter 2 – More Series (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Find the first two non-zero terms of the Maclaurin series expansion of $\cos(2x)$

Tick (✓) **one** box.

[1 mark]

- $1 - x^2$
- $1 - 2x^2$
- $2 - x^2$
- $2 - 2x^2$

(Total 1 marks)

AQA AS June 2025 Paper 1 Q4

2. The first two non-zero terms of the Maclaurin series expansion of $f(x)$ are x and $-\frac{1}{2}x^3$

Which one of the following could be $f(x)$?

Circle your answer.

[1 mark]

- $xe^{\frac{1}{2}x^2}$
- $\frac{1}{2}\sin 2x$
- $x\cos x$
- $(1 + x^3)^{-\frac{1}{2}}$

(Total 1 marks)

AQA A2 June 2019 Paper 1 Q2

3. In this question you may use results from the formulae booklet without proof.

Use the binomial series for $(1 + x)^n$ and the Maclaurin's series for $\sin x$ to find the series

expansion for $\frac{1}{(1 + \sin \theta)^4}$ up to and including the term in θ^3

[4 marks]

(Total 4 marks)

AQA A2 June 2024 Paper 2 Q18

4. Use the method of differences to prove that for $n > 2$

$$\sum_{r=2}^n \frac{4}{r^2 - 1} = \frac{(pn + q)(n - 1)}{n(n + 1)}$$

where p and q are constants to be determined.

(5)

(Total 5 marks)

Edexcel A2 June 2025 Paper 1 Q5

5. Using the Maclaurin series for $\cos 2x$, show that, for small values of x ,

$$\sin^2 x \approx ax^2 + bx^4 + cx^6,$$

where the values of a , b and c are to be given in exact form.

[5]

(Total 5 marks)

OCR MEI A2 June 2019 Paper 1 Q5

6.

(a) Use differentiation to find the first two non-zero terms of the Maclaurin expansion of

$$\ln \left(\frac{1}{2} + \cos x \right).$$

[4]

(b) By considering the root of the equation $\ln \left(\frac{1}{2} + \cos x \right) = 0$ deduce that

$$\pi \approx 3\sqrt{3 \ln \left(\frac{3}{2} \right)}.$$

[3]

(Total 7 marks)

OCR A A2 June 2019 Paper 2 Q10

Solutions

1.

Scheme	Marks	AO
Ticks the 2nd box.	B1	1.1b
	(1 mark)	

Typical solution

$$1 - 2x^2$$

2.

Scheme	Marks	AO
Circles correct answer	B1	2.2a
	(1 mark)	

Typical solution

$$x \cos x$$

3.

Scheme	Marks	AO
Substitutes $\sin \theta$ for x in the first four terms of the correct binomial series with $n = -4$. Condone sign errors.	M1	1.1a
Substitutes in their binomial series the first two terms of the sine series in the first bracket, and the first term of the sine series in subsequent brackets.	M1	3.1a
Obtains the correct series. ISW Condone x used for θ	A1	1.1b
Discards higher powers of θ in a correct, fully simplified, series	A1	2.2a
	(4 marks)	

Typical solution

$$\begin{aligned}
 (1 + \sin \theta)^{-4} &= 1 + (-4) \sin \theta + \frac{(-4)(-5)}{2} \sin^2 \theta + \frac{(-4)(-5)(-6)}{6} \sin^3 \theta + \dots \\
 &= 1 - 4 \left(\theta - \frac{\theta^3}{6} + \dots \right) + 10 \left(\theta - \frac{\theta^3}{6} + \dots \right)^2 \\
 &\quad - 20 \left(\theta - \frac{\theta^3}{6} + \dots \right)^3 + \dots \\
 &= 1 - 4\theta + \frac{2\theta^3}{3} + 10\theta^2 - 20\theta^3 + o(\theta^4) \\
 \frac{1}{(1 + \sin \theta)^4} &= 1 - 4\theta + 10\theta^2 - \frac{58}{3}\theta^3 + \dots
 \end{aligned}$$

4.

Scheme	Marks	AO
$\frac{4}{r^2 - 1} \equiv \frac{4}{(r - 1)(r + 1)} \equiv \frac{A}{r - 1} + \frac{B}{r + 1} \Rightarrow A =$ $\dots, B = \dots$ (NB $A = 2, B = -2$)	M1	3.1a
$r = 2 \Rightarrow \frac{2}{1} - \frac{2}{3}$ $r = 3 \Rightarrow \frac{2}{2} - \frac{2}{4}$ $r = 4 \Rightarrow \frac{2}{3} - \frac{2}{5}$ $r = n - 1 \Rightarrow \frac{2}{n - 2} - \frac{2}{n}$ $r = n \Rightarrow \frac{2}{n - 1} - \frac{2}{n + 1}$	dM1	2.1
$2 + 1 - \frac{2}{n} - \frac{2}{n + 1}$	A1	1.1b
$\frac{3n(n + 1) - 2(n + 1) - 2n}{n(n + 1)} = \frac{3n^2 + 3n - 2n - 2 - 2n}{n(n + 1)} =$ $\frac{3n^2 - n - 2}{n(n + 1)}$	M1	1.1b
$\frac{(3n + 2)(n - 1)}{n(n + 1)} \text{ csO}$	A1	2.2a
	(5)	

(5 marks)

Notes

M1: A complete strategy to write as partial fractions

dM1: Dependent on previous method mark. Completes the method of differences process, must have a minimum substitution of $r = 2$, $r = 3$, $r = n - 1$ and $r = n$. Condone the use of r for n for this mark.

If pairs of terms are not seen in each substitution for r , this mark can be implied by sight of

$$2 + 1 - \frac{2}{n} - \frac{2}{n+1} \text{ following correct partial fractions.}$$

A1: Correct 'fractions' from the beginning and end that do not cancel. Accept exact equivalents. Must be in terms of n

M1: Combines all their 'fractions' (at least two algebraic fractions) over a correct common denominator, leading to a quadratic in the numerator, where brackets may not be expanded. Condone the use of r for n for this mark.

A1: Since the form of the solution is given, only accept $\frac{(3n+2)(n-1)}{n(n+1)}$ and not equivalents, but isw

Do not accept values for p and q written alone.

Note: If they start with $r = 0$, $r = 1$ or $r = 3$, the maximum they can score is M1M0A0M1A0

Similarly, if they use $r = n + 1$ the maximum they can score is M1M0A0M1A0

5.

Scheme	Marks	AO
$\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots$	M1	1.1a
$= 1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \dots$	A1	1.1b
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $= \frac{1}{2} \left(1 - 1 + 2x^2 - \frac{2}{3}x^4 + \frac{4}{45}x^6 + \dots \right)$	M1	3.1a
$= x^2 - \frac{1}{3}x^4 + \frac{2}{45}x^6 + \dots$ so $a = 1$, $b = -\frac{1}{3}$, $c = \frac{2}{45}$	A2,1,0	1.1b
	[5]	

Notes

M1: at least 3 terms correct, or good attempt from 1st principles

A1: Allow unsimplified fractions (without factorials)

6.

Mark scheme (a)

Scheme	Marks	AO
$f(0) = \ln\left(\frac{1}{2} + \cos 0\right) = \ln\left(\frac{3}{2}\right)$	B1	1.1
$\frac{d}{dx} \left(\ln\left(\frac{1}{2} + \cos x\right) \right) = \frac{-\sin x}{\frac{1}{2} + \cos x} \Rightarrow f'(0) = 0$	M1	3.1a
$\frac{d^2 \ln\left(\frac{1}{2} + \cos x\right)}{dx^2} = \frac{-\cos x \left(\frac{1}{2} + \cos x\right) + \sin x(-\sin x)}{\left(\frac{1}{2} + \cos x\right)^2}$ $\dots \Rightarrow f''(0) = -\frac{2}{3}$	A1	1.1
$\ln\left(\frac{1}{2} + \cos x\right) = \ln\left(\frac{3}{2}\right) - \frac{x^2}{3} + \dots$	A1	1.1
	[4]	

Notes

B1: soi

M1: Differentiating using chain rule (or rule for $\ln(f(x))$) and evaluating when $x = 0$. Allow sign error in numerator

A1: Differentiating again using quotient (or product/chain) rule. NB Simplifies to $-\frac{\frac{1}{2} \cos x + 1}{\left(\frac{1}{2} + \cos x\right)^2}$

A1: www

If zero scored then **SC1** for correct expansion

Mark scheme (b)

Scheme	Marks	AO
$\ln\left(\frac{1}{2} + \cos x\right) = 0 \Rightarrow x = \frac{\pi}{3} \text{ (or } -\frac{\pi}{3})$	B1	1.1
$\therefore \ln\left(\frac{3}{2}\right) - \frac{\left(\frac{\pi}{3}\right)^2}{3} \approx 0$	M1	3.1a
$\ln\left(\frac{3}{2}\right) - \frac{\pi^2}{27} \approx 0 \Rightarrow \pi \approx \sqrt{27 \ln\left(\frac{3}{2}\right)} = 3\sqrt{3 \ln\left(\frac{3}{2}\right)}$	A1	3.2a

Notes

B1: Finding either $\pm\pi/3$ as a root. Allow 60° for B1. Ignore other roots

M1: Substituting their root, in radians, into their Maclaurin series and equating (approximately) to 0.

Or equating their expression (approximately) to 0 and rearranging for x :

$$\ln\left(\frac{3}{2}\right) - \frac{x^2}{3} \approx 0 \Rightarrow x \approx \sqrt{3 \ln\left(\frac{3}{2}\right)}$$

$$\frac{\pi}{3} \approx \sqrt{3 \ln\left(\frac{3}{2}\right)} \Rightarrow \pi \approx 3\sqrt{3 \ln\left(\frac{3}{2}\right)}$$

A1: Could see $\pm$ but must be removed by final conclusion. Must use approximately equals symbol (not just equals symbol)