

QQQ – Core Pure 2 – Chapter 2 – More Series (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. Which of the following functions has the fourth term $-\frac{1}{720}x^6$ in its Maclaurin series expansion?

Circle your answer.

[1 mark]

- $\sin x$
- $\cos x$
- e^x
- $\ln(1+x)$

(Total 1 marks)

AQA AS June 2018 Paper 1 Q3

2. In this question you must show detailed reasoning.

Find $\sum_{r=2}^{50} \left(\frac{1}{r-1} - \frac{1}{r+1} \right)$, expressing the answer as an exact fraction.

[3]

(Total 3 marks)

OCR MEI AS October 2020 Paper 1 Q1

3. Show that the Maclaurin series for $\ln(e + 2ex)$ is

$$1 + 2x - 2x^2 + ax^3 - \dots$$

where a is to be determined.

[3 marks]

(Total 3 marks)

AQA AS June 2021 Paper 1 Q7

4. Use the method of differences to show that

$$\sum_{r=1}^n \frac{2}{(r+4)(r+6)} = \frac{n(an+b)}{30(n+5)(n+6)}$$

where a and b are integers to be determined.

(6)

(Total 6 marks)

Edexcel A2 June 2024 Paper 2 Q4

5.

- (a) (i) Given that $f(x) = \sqrt{1+2x}$, find $f'(x)$ and $f''(x)$. [2]

- (ii) Hence, find the first three terms of the Maclaurin series for $\sqrt{1+2x}$. [2]

- (b) Hence, using a suitable value for x , show that $\sqrt{5} \approx \frac{143}{64}$. [2]

(Total 6 marks)

OCR MEI A2 June 2023 Paper 1 Q4

6.

(a) Write down the Maclaurin series of e^x , in ascending power of x , up to and including the term in x^3 **(1)**

(b) Hence, without differentiating, determine the Maclaurin series of

$$e^{(e^x - 1)}$$

in ascending powers of x , up to and including the term in x^3 , giving each coefficient in simplest form. **(5)**

(Total 6 marks)

Solutions

1.

Scheme	Marks	AO
Circles correct answer	B1	1.2
	(1 mark)	

Typical solution

$$\cos x$$

2.

Scheme	Marks	AO
DR $\sum_{r=2}^{50} \left(\frac{1}{r-1} - \frac{1}{r+1} \right)$ $= \left(1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \dots + \frac{1}{47} - \frac{1}{49} + \frac{1}{48} - \frac{1}{50} + \frac{1}{49} - \frac{1}{51} \right)$	M1	2.4
$= \left(1 + \frac{1}{2} - \frac{1}{50} - \frac{1}{51} \right)$	A1*	2.2a
$= \frac{1862}{1275}$	Alcao	1.1
	[3]	

Notes

M1: enough terms to show cancellation clearly

or $\dots + \frac{1}{n-1} - \frac{1}{n+1}$

condone $\dots + \frac{1}{r-1} - \frac{1}{r+1}$

Alcao: dep A1*

3.

Scheme	Marks	AO
Writes the given expression in the form $m + \ln(1 + f(x))$ where m is a non-zero constant (allow unsimplified). Or obtains the first three derivatives of $\ln(e + 2ex)$ in the form $p(q + rx)^n$	M1	3.1a
From an expression of the form $\ln(1 + f(x))$, substitutes $f(x)$ into the Maclaurin series. Condone sign errors and missing brackets. Or substitutes $x = 0$ into their $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$ of the form $p(q + rx)^n$	M1	1.1a
Completes a fully correct argument to reach the required result and correctly calculates a . Do not condone missing brackets in their working.	R1	2.1
	(3 marks)	

Typical solution

$$\begin{aligned}
 \ln(e + 2ex) &= \ln(e(1 + 2x)) \\
 &= \ln e + \ln(1 + 2x) \\
 &= 1 + 2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \dots \\
 &= 1 + 2x - 2x^2 + \frac{8}{3}x^3 - \dots
 \end{aligned}$$

4.

Scheme	Marks	AO
$\frac{2}{(r+4)(r+6)} \equiv \frac{A}{r+4} + \frac{B}{r+6} \Rightarrow A = \dots, B = \dots$	M1	3.1a
$\frac{2}{(r+4)(r+6)} \equiv \frac{1}{r+4} - \frac{1}{r+6}$	A1	1.1b
$\sum_{r=1}^n \frac{2}{(r+4)(r+6)} = \sum_{r=1}^n \frac{1}{r+4} - \frac{1}{r+6}$ $\frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{8} + \frac{1}{7} - \frac{1}{9} + \dots$ $+ \frac{1}{n+2} - \frac{1}{n+4} + \frac{1}{n+3} - \frac{1}{n+5} + \frac{1}{n+4} - \frac{1}{n+6}$	M1	2.1
$= \frac{1}{5} + \frac{1}{6} - \frac{1}{n+5} - \frac{1}{n+6}$	A1	2.2a
$= \frac{1}{5} + \frac{1}{6} - \frac{1}{n+5} - \frac{1}{n+6} = \frac{11(n+5)(n+6) - 30(n+6) - 30(n+5)}{30(n+5)(n+6)}$	M1	1.1b
$= \frac{n(11n+61)}{30(n+5)(n+6)}$	A1	1.1b
	(6)	
	(6 marks)	

Notes

M1: Recognises the need to find partial fractions and applies a correct method leading to finding values for A and B

Allow a slip when finding the constants

A1: Correct partial fractions **seen** at any stage. Not just values for A and B listed

Note: Proof by induction will not score the next 4 marks.

M1: Starts the process of finding terms at the start and at the end, in order to establish the non-cancelling terms

Must have attempted a minimum of $r = 1, r = 2, \dots, r = n - 1$ and $r = n$, this may be implied by their correct non-cancelling terms.

Follow through on their values of A and B . Look for

$$\begin{aligned}
 r = 1 &\rightarrow \frac{A}{5} - \frac{B}{7} & r = 2 &\rightarrow \frac{A}{6} - \frac{B}{8} \\
 r = n - 1 &\rightarrow \frac{A}{n+3} - \frac{B}{n+5} & r = n &\rightarrow \frac{A}{n+4} - \frac{B}{n+6}
 \end{aligned}$$

A1: Correct non-cancelling terms which may be listed separately.

Correct fractions from the beginning and end that do not cancel stated.

M1: Combines 'their' fractions of the form $p + \frac{q}{n+5} + \frac{r}{n+6}$ over a correct common denominator which does not need to be the lowest common denominator and obtains a quadratic expression in the numerator.

A1: Correct answer.

Note: If they start with $r = 0$ the maximum they can score is M1A1M0A0M1A0

5.

Mark scheme (a)

Scheme	Marks	AO
(i) $f'(x) = (1 + 2x)^{-\frac{1}{2}}$	B1	1.1
$f''(x) = -(1 + 2x)^{-\frac{3}{2}}$	B1	1.1
	[2]	
(ii) $f(0) = 1, f'(0) = 1, f''(0) = -1$	M1	1.1
$1 + x - \frac{1}{2}x^2$	A1	1.1
	[2]	

Notes

(a)(ii)

M1: Their $f(0)$, $f'(0)$ and $f''(0)$ evaluated and substituted into Maclaurin

A1: Must come from correct expressions for $f'(x)$, $f''(x)$; cannot come from binomial expansion. Ignore subsequent terms.

Mark scheme (b)

Scheme	Marks	AO
$\sqrt{1 + 2 \times \frac{1}{8}}$ or $1 + \frac{1}{8} - \frac{1}{2} \left(\frac{1}{8}\right)^2$	M1	3.1a
$\Rightarrow \sqrt{5} \approx \frac{143}{64}$	A1	2.2a
	[2]	

Notes

M1: using $x = \frac{1}{8}$ in their expansion

A1: AG

6.

Mark scheme (a)

Scheme	Marks	AO
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ or $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$	B1	1.1b
	(1)	

Notes

B1: Correct series (ignore terms beyond x^3).

Mark scheme (b)

Scheme	Marks	AO
Version 1 $e^{(e^x-1)} = e^{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots-1}$ or Version 2 $e^{(e^x-1)} = 1 + (e^x - 1) + \frac{(e^x - 1)^2}{2!} + \frac{(e^x - 1)^3}{3!} + \dots$	M1	1.1b

Version 1.1 $= 1 + \left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right) + \frac{1}{2} \left(x + \frac{x^2}{2!} + \dots\right)^2 + \frac{1}{6}(x + \dots)^3 + \dots$ Or Version 1.2 $e^{x+\frac{x^2}{2}+\frac{x^3}{6}} = e^x \times e^{\frac{x^2}{2}} \times e^{\frac{x^3}{6}}$ $= \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) \left(1 + \frac{x^2}{2} + \dots\right) \left(1 + \frac{x^3}{6} + \dots\right)$ Or Version 2.1 $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \dots$ $= \frac{1}{3} + \frac{1}{6}e^{3x} + \frac{1}{2}e^x$ $= \frac{1}{3} + \frac{1}{6} \left(1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{6}\right) + \frac{1}{2} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)$ Or Version 2.2 $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \dots$ $= 1 + \left(x + \frac{x^2}{2} + \frac{x^3}{6}\right) + \frac{1}{2} \left[\left(1 + 2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{6}\right) - 2 \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) + 1\right]$ $+ \frac{1}{6} \left[\left(1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{6}\right) - 3 \left(1 + 2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{6}\right) + 3 \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) - 1\right]$	M1	3.1a
$= 1 + x + \left(\frac{1}{2} + \frac{1}{2}\right)x^2 + \left(\frac{1}{6} + \frac{1}{2} \times 2 \times \frac{1}{2} + \frac{1}{6}\right)x^3 + \dots$	dM1	1.1b
$= 1 + x + x^2 + \frac{5}{6}x^3 + \dots$	A1 A1	1.1b 2.1
	(5)	
	(6 marks)	

Notes

M1: Correctly applies the exponential Maclaurin expansion at least once, either to the base exponent or in the index. Allow 2 for 2! and 6 for 3! in the cube term. Follow through on their series seen in (a)

M1: A complete attempt to use the exponential Maclaurin series to produce a cubic expression in terms of x only. Allow if the 3! is incorrect for this mark, but a polynomial in x must have been achieved. Condone a slip with one term. Follow through on their series seen in (a)

dM1: Dependent on previous method mark only. Expands the brackets and gathers terms (not necessarily fully simplified, but should have a single term for each power).

A1: Any two correct from coefficients of x , x^2 and x^3 , need not be simplified.

A1: Fully correct answer with simplified terms.

NB: Question instructs to use standard Maclaurin series, so use of differentiation scores no mark.

Special case: Using $e^{(e^x-1)} = e^{e^x} \times e^{-1}$ can score M1M1M0A0A0 for using Maclaurin series e^{e^x} and then on e^x , e^{2x} , e^{3x}