

QQQ – Core Pure 1 – Chapter 9 – Vectors (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Find, to the nearest degree, the angle between the vectors $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix}$. **[3]**

(Total 3 marks)

OCR MEI AS June 2018 Paper 1 Q2

2. The line l has equation

$$\frac{x+2}{1} = \frac{y-5}{-1} = \frac{z-4}{-3}$$

The plane Π has equation

$$\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = -7$$

Determine whether the line l intersects Π at a single point, or lies in Π , or is parallel to Π without intersecting it. **(5)**

(Total 5 marks)

Edexcel AS June 2019 Paper 1 Q4

3. An engineer detects a source of water below the surface of the ground. The engineer models the situation relative to a fixed origin O .

In the model

- the surface of the ground is a plane Π with equation $x - 2y + 8z = 1$
- the source of water is at a point W with coordinates $(6, -2, -4)$

where the units are metres.

- (a) Use the model to determine the shortest distance from W to the surface of the ground. (2)
- (b) By considering the model, comment on whether the answer to part (a) is reliable, giving a reason for your answer. (1)

To access the water, a hole is drilled, in a straight line, from a point P on the surface of the ground to W .

Given that the length of the hole needs to be as short as possible,

- (c) determine the coordinates of P , according to the model. (3)

Given that the actual length of the hole drilled is 2.52 metres,

- (d) use the answer to part (a) to evaluate the model, giving a reason for your answer. (1)

(Total 7 marks)

Edexcel AS June 2025 Paper 1 Q9

4. The points $A(5, -4, 6)$ and $B(6, -6, 8)$ lie on the line L . The point C is $(15, -5, 9)$.

- (a) D is the point on L that is closest to C .

Find the coordinates of D . **[6 marks]**

- (b) Hence find, in exact form, the shortest distance from C to L . **[2 marks]**

(Total 8 marks)

AQA A2 June 2019 Paper 1 Q10

Solutions

1.

Scheme	Marks	AO
$\cos \theta = \frac{1 \times (-2) + 0 \times 3 + (-2) \times (-3)}{\sqrt{5}\sqrt{22}} = 0.381 \dots$	M1	1.1a
	A1	1.1
$\theta = 68^\circ$ to nearest degree	A1	1.1
	[3]	

Notes

M1: Use of $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$

A1: (1st) 0.381 ... or $\frac{4}{\sqrt{5}\sqrt{22}}$

A1: (2nd) must be nearest degree
allow unsupported correct answers

2.

Scheme	Marks	AO
$(\mathbf{r} =) \begin{pmatrix} -2 + \lambda \\ 5 - \lambda \\ 4 - 3\lambda \end{pmatrix} \text{ or } \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix} \text{ (oe)}$	M1	1.1b
So meet if $\begin{pmatrix} -2 + \lambda \\ 5 - \lambda \\ 4 - 3\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = -7 \Rightarrow (-2 + \lambda) \times 1 + (5 - \lambda) \times$ $-2 + (4 - 3\lambda) \times 1 = -7$	M1 A1	3.1a 1.1b
$\Rightarrow 0\lambda - 8 = -7 \Rightarrow -8 = -7$ a contradiction so no intersection	A1ft	2.3
Hence l is parallel to l' but not in it.	A1cso	3.2a
	(5)	
	(5 marks)	

Notes

M1: Forms a parametric form for the line. Allow one slip.

M1: Substitutes into the equation of the plane to an equation in λ . May use Cartesian form of plane to substitute into.

A1: Correct equation in λ

A1ft: Simplifies and derives a contradiction and deduces line and plane do not meet. Follow through in their initial equation in λ so

- contradiction so no intersection if λ disappears and constants unequal
- line lies in plane if a tautology is arrived at
- meet in a point if a solution for λ is found.

But do not allow for incorrect simplification from a correct initial equation in λ

Note that a miscopy/misread of 7 instead of -7 can therefore score a maximum of M1M1A0A1A0.

A1cso: Correct deduction from correct working. This may be seen two separate statements in their working. You may see attempts at showing the line is parallel before/after deducing there is no intersection.

Alternative (Alt 1)

Note that some may attempt a mix of the main scheme and Alt 1. Mark under main scheme unless Alt 1 would score higher.

Scheme	Marks	AO
$\begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 1 \times 1 + (-1) \times (-2) + (-3) \times 1 = 0$	M1	3.1a
Hence l is parallel to Π	A1	1.1b
$(-2, 5, 4)$ on l , but $(1)(-2) + (-2)(5) + 1(4) = -8$	M1	1.1b
$-8 \neq -7$ so $(-2, 5, 4)$ is not on the plane.	A1ft	2.3
Hence l is (parallel to Π but) not in the plane.	A1cso	3.2a
	(5)	

M1: Attempts the dot product between the two direction vectors.

A1: Shows dot product is zero and makes the correct deduction that line is parallel to plane.

M1: Finds a point on l and substitutes into the equation of Π (vector or Cartesian)

A1ft: Simplifies and derives a contradiction – follow through their equation, so if arrive at a tautology, they should deduce the line is in the plane.

A1cso: Correct deduction from correct working but may be split across working.

Alternative (Alt 2)

Scheme	Marks	AO
Attempts to solve $\frac{x+2}{1} = \frac{y-5}{-1} = \frac{z-4}{-3}$ and $x - 2y + z = -7$ simultaneously – eliminates one variable for M mark.	M1	3.1a
e.g. $y = -(x+2) + 5 = -x + 3 \Rightarrow x - 2(-x + 3) + z = -7 \Rightarrow$ $3x + z = -1$ (oe)	A1	1.1b
Solves reduced equations, e.g. $-3(x+2) = z-4 \Rightarrow 3x + z = -2$ and $3x + z = -1 \Rightarrow (3x + z) - (3x + z) = -2 - (-1)$	M1	1.1b
$\Rightarrow 0 = -1$ a contradiction so no intersection	A1ft	2.3
Hence l is parallel to Π but not in it.	A1cso	3.2a
	(5)	

M1: Attempts to solve the Cartesian equation of the line and plane, using the plane equation to eliminate one variable for the M.

A1: Correct elimination of their chosen variable. (E.g. may see $3 - 3y + z = -7$ or $-2x - 2y - 2 = -7$ etc)

M1: Solves the reduced equations in two variables...

A1ft: ... and derives a contradiction/line and plane do not meet. Follow through their result, so may reach a tautology and deduce lies in plane, or find single solution and deduce meet in a point.

A1cso: Correct deduction from correct working.

3.

Mark scheme (a)

Scheme	Marks	AO
$\frac{(1 \times 6) + (-2 \times -2) + (8 \times -4) \pm 1}{\sqrt{1^2 + (-2)^2 + 8^2}}$	M1	3.4
$\frac{23}{\sqrt{69}}$ or $\frac{1}{3}\sqrt{69}$ or awrt 2.77	A1	1.1b
	(2)	

Notes

M1: Uses the shortest distance formula $\frac{(1 \times 6) + (-2 \times -2) + (8 \times -4) \pm 1}{\sqrt{1^2 + (-2)^2 + 8^2}}$ condoning a sign slip

with d and no modulus for this mark.

May find the answer to (c) first and then find the distance. It must be a correct method, scoring at least M1M1 in (c)

A1: Correct exact answer or awrt 2.77 isw

Mark scheme (b)

Scheme	Marks	AO
e.g. not reliable as The source of the water will not be at a single point The water level will vary The ground may not be flat	B1	3.2b
	(1)	

Notes

B1: See scheme

Note: the position of the water may change is B0

Mark scheme (c)

Scheme	Marks	AO
$\mathbf{r} = \begin{pmatrix} 6 \\ -2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix}$ and $(6 + \lambda) - 2(-2 - 2\lambda) + 8(-4 + 8\lambda) = 1 \Rightarrow \lambda = \dots \left\{ \frac{1}{3} \right\}$ Or Their $\frac{1}{3}\sqrt{69}$ divided by $\sqrt{1^2 + (-2)^2 + (8)^2}$ leading to a value for λ	M1	3.1b
$\mathbf{r} = \begin{pmatrix} 6 \\ -2 \\ -4 \end{pmatrix} + \text{their } \frac{1}{3} \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} = \dots$	dM1	1.1b
$\left(\frac{19}{3}, -\frac{8}{3}, -\frac{4}{3} \right)$ or $\left(6\frac{1}{3}, -2\frac{2}{3}, -1\frac{1}{3} \right)$ only	A1	1.1b

	(3)	
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Notes

(c) This may be seen in part (a) to score any marks at least the final coordinate must be stated in (c) or 'see (a)'

M1: A complete method to find the value of λ

dM1: Dependent on the previous method mark. Uses their value of λ in the equation of the line

$$\mathbf{r} = \begin{pmatrix} 6 \\ -2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} = \dots \text{ to find a coordinate}$$

A1: Correct coordinate

Mark scheme (d)

Scheme	Marks	AO
Compares their answer to (a) with 2.52, must give some idea about the difference between the values and draws an appropriate conclusion Answer from (a) • Less than 2.2 or more than 2.8 should be saying a poor model • Between 2.2 and 2.4 or between 2.6 and 2.8 either good or bad model • Between 2.4 and 2.6 should be saying a good model	B1	3.5a
	(7 marks)	

Notes

B1: See scheme, must have an answer to (a) so score this mark

4.

Mark scheme (a)

Scheme	Marks	AO
Obtains an equation of L . Condone one error in their direction vector. Condone lack of " $\mathbf{r} =$ ", PI by correct $\mathbf{v}$	M1	1.1a

Obtains a correct equation of L . Condone lack of “ $\mathbf{r} =$ ”, PI by correct $\mathbf{v}$	A1	1.1b
Obtains their correct general vector from line to C	B1F	3.1a
Finds scalar product of their $\mathbf{v}$ and their $\overrightarrow{AB}$	M1	3.1a
Solves to find the correct μ for their equation.	A1F	1.1b
Finds correct D	A1	3.2a

Typical solution

$$\mathbf{r} = \begin{bmatrix} 5 \\ -4 \\ 6 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} -10 + \mu \\ 1 - 2\mu \\ -3 + 2\mu \end{bmatrix}$$

$$0 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -10 + \mu \\ 1 - 2\mu \\ -3 + 2\mu \end{bmatrix}$$

$$-10 + \mu - 2 + 4\mu - 6 + 4\mu = 0$$

$$\mu = 2$$

$$D = (7, -8, 10)$$

Mark scheme (b)

Scheme	Marks	A0
Obtains their components of $\overrightarrow{CD}$, must have their correct magnitude, but ignore sign. Allow one error.	M1	1.1a
Obtains their correct CD , in exact form.	A1F	1.1b
	(8 marks)	

Typical solution

$$\overrightarrow{CD} = \begin{pmatrix} -8 \\ -3 \\ 1 \end{pmatrix}$$

$$CD = \sqrt{74}$$