

QQQ – Core Pure 1 – Chapter 9 – Vectors (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The line L has vector equation

$$\mathbf{r} = \begin{bmatrix} 4 \\ -7 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -9 \\ 1 \\ 3 \end{bmatrix}$$

Give the equation of L in Cartesian form.

Tick (✓) **one** box.

[1 mark]

- $\frac{x+4}{-9} = \frac{y-7}{1} = \frac{z}{3}$
- $\frac{x-4}{-9} = \frac{y+7}{1} = \frac{z}{3}$
- $\frac{x+9}{4} = \frac{y-1}{-7}, z = 3$
- $\frac{x-9}{4} = \frac{y+1}{-7}, z = 3$

(Total 1 marks)

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2. A plane has equation $\mathbf{r} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 7$

A line has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Calculate the acute angle between the line and the plane.

Give your answer to the nearest 0.1°

[3 marks]

(Total 3 marks)

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3. The plane $x + 2y + cz = 4$ is perpendicular to the plane $2x - cy + 6z = 9$, where c is a constant. Find the value of c .

[3]

(Total 3 marks)

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4. Two submarines are travelling on different straight lines.

The two lines are described by the equations

$$\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \quad \text{and} \quad \frac{x-5}{4} = \frac{y}{2} = 4-z$$

(a) (i) Show that the two lines intersect. [3 marks]

(ii) Find the position vector of the point of intersection. [1 mark]

(b) Tracey says that the submarines will collide because there is a common point on the two lines.

Explain why Tracey is not necessarily correct. [1 mark]

(c) Calculate the acute angle between the lines

$$\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \quad \text{and} \quad \frac{x-5}{4} = \frac{y}{2} = 4-z$$

Give your angle to the nearest 0.1° [3 marks]

(Total 8 marks)

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5. The line through points $A(8, -7, -2)$ and $B(11, -9, 0)$ is denoted by L_1 .

(a) Find a vector equation for L_1 . [2]

(b) Determine whether the point $(26, -19, -14)$ lies on L_1 . [2]

The line L_2 passes through the origin, O , and intersects L_1 at the point C . The lines L_1 and L_2 are perpendicular.

(c) By using the fact that C lies on L_1 , find a vector equation for L_2 . [4]

(d) Hence find the shortest distance from O to L_1 . [2]

(Total 10 marks)

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Solutions

1.

Scheme	Marks	AO
Ticks the 2 nd box.	B1	1.1b
	(1 mark)	

Typical solution

$$\frac{x-4}{-9} = \frac{y+7}{1} = \frac{z}{3}$$

2.

Scheme	Marks	AO
Finds scalar (or vector) product of the correct vectors PI by seeing AWRT 35°	M1	1.1a
Divides their scalar product (or magnitude of vector product) of their vectors by product of their magnitudes PI by seeing AWRT 35°	M1	1.1a
Deduces the correct angle, correct to at least 1dp	A1	2.2a
	(3 marks)	

Typical solution

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 2$$

Moduli of vectors are $\sqrt{2}$ and $\sqrt{3}$

Let α be angle between normal & line

$$\cos \alpha = \frac{2}{\sqrt{6}}$$

Angle between plane & line

$$= 90 - \alpha = 54.7^\circ$$

3.

Scheme	Marks	AO
$(\mathbf{i} + 2\mathbf{j} + c\mathbf{k}) \cdot (2\mathbf{i} - c\mathbf{j} + 6\mathbf{k}) = 0$	M1	1.1a
$\Rightarrow 2 - 2c + 6c = 0$	A1	1.1b
$\Rightarrow c = -\frac{1}{2}$	A1	1.1b
	[3]	

Notes

M1: scalar product = 0

4.

Mark scheme (a)

Scheme	Marks	AO
(i) Forms one equation by substituting two of $x = 2 + 5\lambda$, $y = -1 + 3\lambda$, $z = 4 - 2\lambda$ into the equation of the second line. Or introduces a second parameter (eg μ) to form at least two equations in λ and μ , e.g. $2 + 5\lambda = 4\mu + 5$, $-1 + 3\lambda = 2\mu$, $4 - 2\lambda = 4 - \mu$	M1	1.1a
Solves their equation(s) to obtain the correct value of λ (or μ).	A1	1.1b
Completes a correct argument to conclude that the lines intersect. eg shows that $\lambda = -1$ satisfies another equation formed by substituting a different combination of x , y and z Or shows that $\lambda = -1$ and $\mu = -2$ satisfy a third equation. Or shows that $(-3, -4, 6)$ lies on both lines.	R1	2.1
	(3)	
(ii) Obtains $\begin{bmatrix} -3 \\ -4 \\ 6 \end{bmatrix}$ Accept $-3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ Condone $(-3, -4, 6)$	B1	1.1b

	(1)	
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Typical solution

(i)

$$\frac{2 + 5\lambda - 5}{4} = \frac{-1 + 3\lambda}{2}$$

$$2(5\lambda - 3) = 4(3\lambda - 1)$$

$$10\lambda - 12\lambda = -4 + 6$$

$$-2\lambda = 2$$

$$\lambda = -1$$

$$\frac{2 + 5\lambda - 5}{4} = 4 - (4 - 2\lambda)$$

$$5\lambda - 3 = 4 \times 2\lambda$$

$$-3 = 8\lambda - 5\lambda$$

$$-3 = 3\lambda$$

$$\lambda = -1$$

same value of λ $\therefore$ the lines intersect

(ii)

$$\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} - 1 \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \\ 6 \end{bmatrix}$$

Mark scheme (b)

Scheme	Marks	AO
States a correct explanation.	B1	3.2a
	(1)	

Typical solution

The submarines might not be at the intersection point at the same time.

Mark scheme (c)

Scheme	Marks	AO
Calculates the scalar product of the two direction vectors. Allow one error in each direction vector.	M1	1.1a

Calculates the product of the direction vector magnitudes.	M1	1.1a
Obtains correct angle. Condone 0.13 (radians). Accept 7.61° or better.	A1	1.1b
	(3)	
	(8 marks)	

Typical solution

$$\begin{aligned} & \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix} \\ &= 5 \times 4 + 3 \times 2 + (-2) \times (-1) = 28 \\ & \sqrt{5^2 + 3^2 + (-2)^2} \times \sqrt{4^2 + 2^2 + (-1)^2} \\ &= \sqrt{38 \times 21} = \sqrt{798} \end{aligned}$$

$$\cos \theta = \frac{28}{\sqrt{798}}$$

$$\theta = 7.6^\circ \text{ (1dp)}$$

5.

Mark scheme (a)

Scheme	Marks	AO
$\overrightarrow{AB} = \begin{pmatrix} 11 \\ -9 \\ 0 \end{pmatrix} - \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$	M1	1.1
$\mathbf{r} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$	A1	1.1
	[2]	

Notes

M1: Subtracting the position vectors of A and B (in either order). Calculation or answer is enough for M1
Allow M1 for a row vector i.e. (3, -2, 2)

A1: Must be “ $\mathbf{r} =$ ” (or “ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ ”)

Allow $\mathbf{r}$ or $\mathbf{L}_1$ but not L_1 (i.e. allow $\mathbf{r}$ not to be underlined as a vector, but if L_1 used this must be indicated as a vector.

$$\mathbf{r} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} \text{ or } \begin{pmatrix} 11 \\ -9 \\ 0 \end{pmatrix} \pm \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$$

Note – could use other “starting points”

Must be column vectors here

Mark scheme (b)

Scheme	Marks	AO
$8 + 3\lambda = 26$ or $-7 - 2\lambda = -19$ or $-2 + 2\lambda = -14$	M1	1.1
(x gives $\lambda = 6$ or y gives or $\lambda = 6$) [but] z gives $\lambda = -6$ so (26, -19, -14) does not lie on the line	A1FT	2.2a
	[2]	

Notes

M1: Writing down a correct equation for any component of their declared line.

Note – values of λ will depend on the “starting point” used in part (a) and different multiples of the direction vector.

A1FT: Finding a correct inconsistency and reaching correct conclusion. Not all values need be given but values given must be correct. FT their declared line.

Could see eg $\lambda = 6 \Rightarrow (y = -19 \text{ but}) z = 10$ which is not -14

Do not need to see the word “inconsistent” or explicit comparison. Finding two different values of λ and then stating does not lie on line is enough

Alternate method

Scheme	Marks
$\begin{pmatrix} 18 \\ -12 \\ -12 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$	M1
But $\begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ is not parallel to $\begin{pmatrix} 18 \\ -12 \\ -12 \end{pmatrix}$ so the point does not lie on the line	A1FT

M1: Rearranging vector equation

A1FT: Or finding λ inconsistency as before

A0 for “there is no value of λ that works” without justification

Mark scheme (c)

Scheme	Marks	AO
$\overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ [for some particular value of λ .]	B1FT	3.1a
$\left[\overrightarrow{OC} = \mu \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right]$ and $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = 0$	B1FT	2.1
$3a - 2b + 2c = 0$ $3(8 + 3\lambda) - 2(-7 - 2\lambda) + 2(-2 + 2\lambda) = 0$	M1	1.1
$\Rightarrow \lambda = -2 \Rightarrow \overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} - 2 \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$ so the equation of L_2 is $\mathbf{r} = \mu \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$	A1	2.2a
	[4]	

Notes

B1FT: Can be embedded or implied. Can be same or different symbol as parameter in (a).

B1FT: Condition for perpendicularity stated.

Don't have to see an equation for OC in terms of μ here.

M1: Forming a correct dot product and using it with the other definition of $\overrightarrow{OC}$ to form an equation in the parameter.

μ may or may not be present here, but must be dealt with appropriately for the A mark.

A1: Condone use of the same parameter as L_1 .

Could see eg $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$

BOD lack of “ $\mathbf{r} =$ ” (or “ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ ”) in this part if has already been penalised in part (a). If full marks

awarded in part (a) then must have correct notation here for full marks.

Alternative Method

Scheme	Marks
$\overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ for some particular value of λ .	B1FT
$\overrightarrow{OC} \cdot \mathbf{b}_{L_1} = 0$	B1FT
$\therefore \left(\begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \right) \cdot \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ $= 24 + 14 - 4 + (9 + 4 + 4)\lambda = 0$	M1
$\Rightarrow \lambda = -2 \Rightarrow \overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} - 2 \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$ so the equation of L_2 is $\mathbf{r} = \mu \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$	A1

B1FT: Can be embedded or implied. Can be same or different symbol as parameter in (a).

B1FT: Condition for perpendicularity stated.

M1: Forming a correct dot product and using it to form an equation in the parameter.

$$\therefore \begin{pmatrix} 8 + 3\lambda \\ -7 - 2\lambda \\ -2 + 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = 24 + 9\lambda + 14 + 4\lambda - 4 + 4\lambda = 0$$

(Corrected from the printed mark scheme: the printed guidance has $10 + 2\lambda$ as the third component and $+20$ in the expansion; the third component of $\overrightarrow{OC}$ is $-2 + 2\lambda$, giving -4 .)

A1: Could see eg $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$

Condone presence of zero vector as first point

Condone use of the same parameter as L_1 .

BOD lack of " $\mathbf{r} =$ " (or " $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ ") in this part if has already been penalised in part (a). If full marks

awarded in part (a) then must have correct notation here for full marks.

Alternative Method 2

Scheme	Marks
$\overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ for some particular value of λ .	B1FT

Distance $ \overrightarrow{OC} ^2$ is given by: $ \overrightarrow{OC} ^2 = (8 + 3\lambda)^2 + (-7 - 2\lambda)^2 + (-2 + 2\lambda)^2$ $= 64 + 48\lambda + 9\lambda^2 + 49 + 28\lambda + 4\lambda^2 + 4 - 8\lambda + 4\lambda^2$ $= 17\lambda^2 + 68\lambda + 117$	M1
Equating the derivative to 0 gives $34\lambda + 68 = 0 \Rightarrow \lambda = -2$	M1
$\Rightarrow \lambda = -2 \Rightarrow \overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} - 2 \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$ so the equation of L_2 is $\mathbf{r} = \mu \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$	A1

B1FT: Can be embedded or implied. Can be same or different symbol as parameter in (a).

M1: Using the fact that minimum distance is the perpendicular distance.

A1: Condone use of the same parameter as L_1 .

Condone presence of zero vector as first point

Could see eg $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$

BOD lack of " $\mathbf{r} =$ " (or " $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ ") in this part if has already been penalised in part (a). If full marks

awarded in part (a) then must have correct notation here for full marks.

Mark scheme (d)

Scheme	Marks	AO
Because C lies on L_1 and L_2 and OC is perpendicular to L_1 , OC must be the shortest route from O to L_1 .	M1	3.1a
$ \overrightarrow{OC} = \sqrt{2^2 + (-3)^2 + (-6)^2} = \sqrt{49} = 7$ so shortest distance from O to L_1 is 7 units.	A1	1.1
	[2]	

Notes

M1: Can be implied by sight of $|\overrightarrow{OC}|$. Or attempt at evaluating $|\overrightarrow{OC}|$

If just an evaluation seen need to see the vector identified as C in previous part (could be from equation).
i.e. do not allow method mark for modulus of a vector which is not C.

A1: Follow through sign errors on coordinates of C