

QQQ – Core Pure 1 – Chapter 8 – Proof by Induction (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_1 = 1$$

$$u_{n+1} = u_n + 3n$$

Prove by induction that for all integers $n \geq 1$

$$u_n = \frac{3}{2}n^2 - \frac{3}{2}n + 1$$

[4 marks]

(Total 4 marks)

2. Astrid is solving this mathematics problem:

The series S_n is defined by

$$S_n = 2 + 4 + 6 + \dots + 2n \quad (n \in \mathbb{Z}, n \geq 1)$$

Prove by induction that

$$S_n = n(n + 1)$$

Astrid's solution is as follows:

Assume the result is true for $n = k$

Then

$$S_k = k(k + 1)$$

$$S_{k+1} = S_k + 2(k + 1)$$

$$S_{k+1} = k(k + 1) + 2(k + 1)$$

$$S_{k+1} = (k + 2)(k + 1)$$

$$S_{k+1} = (k + 1)((k + 1) + 1)$$

So the result is also true for $n = k + 1$

The result is true for $n = 1$

It is true for $n = k$, and also true for $n = k + 1$

Hence, by induction $S_n = n(n + 1)$ for all integers $n \geq 1$

- (a) (i) Chloe says that Astrid missed out an essential part of the proof, which could have been written at the start.

Explain what Astrid missed out.

[1 mark]

- (ii) Write down the working that Astrid missed out.

[1 mark]

- (b) (i) One statement in the last three lines of Astrid's solution is written incorrectly.

Which statement is written incorrectly?

Tick (✓) **one** box.

[1 mark]

- The result is true for $n = 1$

- It is true for $n = k$, and also true for $n = k + 1$
- Hence, by induction $S_n = n(n + 1)$ for all integers $n \geq 1$

(ii) Write out a correct statement which should replace the incorrect statement identified in part (b)(i) **[1 mark]**

(Total 4 marks)

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3. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1}$$

(5)

(Total 5 marks)

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4. Prove by induction that $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{pmatrix}$ for all positive integers n . **[6]**

(Total 6 marks)

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5. Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 2^{n+2} + 3^{2n+1}$$

is divisible by 7

(6)

(Total 6 marks)

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