QQQ – Core Pure 1 – Chapter 7 – Linear Transformations (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1.

(a) A matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$.

Describe the transformation represented by $\mathbf{M}$. **[2]**

(b) Write down the 2×2 matrix that represents a rotation of 90° anticlockwise about the origin. **[1]**

(c) Write down the 3×3 matrix that represents a reflection in the x–z plane. **[1]**

(Total 4 marks)

2. The matrix $\mathbf{M}$ is $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(a) **(i)** Calculate $\det \mathbf{M}$. **[1]**

(ii) State two geometrical consequences of this value for the transformation associated with $\mathbf{M}$. **[2]**

(b) Describe fully the transformation associated with $\mathbf{M}$. **[1]**

(Total 4 marks)

3. The transformation U, represented by the 2×2 matrix $\mathbf{P}$, is a rotation through $90°$ anticlockwise about the origin.

(a) Write down the matrix $\mathbf{P}$. **(1)**

The transformation V, represented by the 2×2 matrix $\mathbf{Q}$, is a reflection in the line $y = -x$.

(b) Write down the matrix $\mathbf{Q}$. **(1)**

Given that U followed by V is transformation T, which is represented by the matrix $\mathbf{R}$,

(c) express $\mathbf{R}$ in terms of $\mathbf{P}$ and $\mathbf{Q}$, **(1)**

(d) find the matrix $\mathbf{R}$, **(2)**

(e) give a full geometrical description of T as a single transformation. **(2)**

(Total 7 marks)

4.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find $\det \mathbf{A}$. **(1)**

(b) Find $\mathbf{A}^{-1}$. **(2)**

The triangle R is transformed to the triangle S by the matrix $\mathbf{A}$.
Given that the area of triangle S is 72 square units,

(c) find the area of triangle R. **(2)**

The triangle S has vertices at the points $(0,\ 4)$, $(8,\ 16)$ and $(12,\ 4)$.

(d) Find the coordinates of the vertices of R. **(4)**

(Total 9 marks)