QQQ – Core Pure 1 – Chapter 7 – Linear Transformations (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1.

(a) A matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$.

Describe the transformation represented by $\mathbf{M}$. **[2]**

(b) Write down the 2×2 matrix that represents a rotation of 90° anticlockwise about the origin. **[1]**

(c) Write down the 3×3 matrix that represents a reflection in the x–z plane. **[1]**

(Total 4 marks)

2. The matrix $\mathbf{M}$ is $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(a) **(i)** Calculate $\det \mathbf{M}$. **[1]**

(ii) State two geometrical consequences of this value for the transformation associated with $\mathbf{M}$. **[2]**

(b) Describe fully the transformation associated with $\mathbf{M}$. **[1]**

(Total 4 marks)

3. The transformation U, represented by the 2×2 matrix $\mathbf{P}$, is a rotation through $90°$ anticlockwise about the origin.

(a) Write down the matrix $\mathbf{P}$. **(1)**

The transformation V, represented by the 2×2 matrix $\mathbf{Q}$, is a reflection in the line $y = -x$.

(b) Write down the matrix $\mathbf{Q}$. **(1)**

Given that U followed by V is transformation T, which is represented by the matrix $\mathbf{R}$,

(c) express $\mathbf{R}$ in terms of $\mathbf{P}$ and $\mathbf{Q}$, **(1)**

(d) find the matrix $\mathbf{R}$, **(2)**

(e) give a full geometrical description of T as a single transformation. **(2)**

(Total 7 marks)

4.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find $\det \mathbf{A}$. **(1)**

(b) Find $\mathbf{A}^{-1}$. **(2)**

The triangle R is transformed to the triangle S by the matrix $\mathbf{A}$.
Given that the area of triangle S is 72 square units,

(c) find the area of triangle R. **(2)**

The triangle S has vertices at the points $(0,\ 4)$, $(8,\ 16)$ and $(12,\ 4)$.

(d) Find the coordinates of the vertices of R. **(4)**

(Total 9 marks)

Solutions

1. OCR A A2 June 2025 Paper 1 Q1

Mark scheme (a)

Scheme	Marks	AO
Enlargement, scale factor 5, centre at the origin	B2	1.2 1.1
	[2]	

Notes

B2: for all three correct components, **B1** for any two

Accept "enlarge" and accept "stretch" but only if specified in both x and y directions

For 'scale factor' accept just 'factor' or 'SF' but not just '5'

For 'origin' accept O or $(0, 0)$ allow omission of 'centre' provided intention is clear (e.g. 'about O')

If more than two transformations given, then **B0** (unless this is two stretches in both x and y directions)

Mark scheme (b)

Scheme	Marks	AO
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1	1.1
	[1]	

Notes

B1: cao

Mark scheme (c)

Scheme	Marks	AO
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	B1	1.1
	[1]	

Notes

B1: cao – only penalise the lack of a bracket around the entries once in parts **(b)** and **(c)**

2. OCR MEI AS October 2020 Paper 1 Q4

Mark scheme (a)

Scheme	Marks	AO
(i) $\det \mathbf{M} = 1$	B1	1.1
	[1]	
(ii) It preserves volume	B1	1.1
and orientation	B1	1.1
	[2]	

Notes

(a)(i)

B1: BC

Mark scheme (b)

Scheme	Marks	AO
Rotation of $90°$ [anticlockwise] about Oz.	B1	1.1
	[1]	

3. Edexcel FP1 January 2013 Q4

Mark scheme (a)

Scheme	Marks
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1
	(1)

Notes

(a) and (b) Signs must be clear for B marks.

Mark scheme (b)

Scheme	Marks

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1
	(1)

Notes

(a) and (b) Signs must be clear for B marks.

Mark scheme (c)

Scheme	Marks
$\mathbf{R = QP}$	B1
	(1)

Notes

(c) Accept $\mathbf{QP}$ or their 2x2 matrices in the correct order only for B1.

Mark scheme (d)

Scheme	Marks
$\mathrm{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1 cao
	(2)

Notes

(d) M for their $\mathbf{QP}$ where answer involves ± 1 and 0 in a 2x2 matrix, A for correct answer only.

Mark scheme (e)

Scheme	Marks
Reflection in the y axis	B1 B1
	(2)
	[7]

Notes

(e) First B for Reflection, Second B for 'y axis' or '$x = 0$'. Must be single transformation. Ignore any superfluous information.

4. Edexcel FP1 January 2011 Q8

Mark scheme (a)

Scheme	Marks
$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$	
$\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$ $\underline{4}$	B1
	(1)

Mark scheme (b)

Scheme	Marks
$\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ $\frac{1}{\det \mathbf{A}}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	M1
$\frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	A1
	(2)

Mark scheme (c)

Scheme	Marks
$\text{Area}(R) = \frac{72}{4} = \underline{18}\ (\text{units})^2$ $\frac{72}{\text{their } \det \mathbf{A}}$ *or* $72(\text{their } \det \mathbf{A})$	M1
$\underline{18}$ or ft answer.	A1ft
	(2)

Mark scheme (d)

Scheme	Marks
$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1}\mathbf{AR} = \mathbf{A}^{-1}\mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1}\mathbf{S}$	

$\mathbf{R} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$ *At least one attempt to apply* $\mathbf{A}^{-1}$ *by any of the three vertices in* $\mathbf{S}$.	M1
$= \frac{1}{4}\begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$	
$= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$ *At least one correct* ***column*** *o.e.*	A1ft
At least two correct ***columns*** *o.e.*	A1
Vertices are $(2,\ 2)$, $(14,\ 10)$ and $(11,\ 5)$. *All three* ***coordinates*** *correct.*	A1
	(4)
	[9]