QQQ – Core Pure 1 – Chapter 7 – Linear Transformations (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

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1. Find two invariant points under the transformation given by $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ **[2 marks]**

(Total 2 marks)

2.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Describe fully the single geometric transformation A represented by the matrix $\mathbf{A}$. **(2)**

$$\mathbf{B} = \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

The transformation B is represented by the matrix $\mathbf{B}$.

The transformation A followed by the transformation B is the transformation C, which is represented by the matrix $\mathbf{C}$.

To determine matrix $\mathbf{C}$, a student attempts the following matrix multiplication.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

(b) State the error made by the student. **(1)**

(c) Determine the correct matrix $\mathbf{C}$. **(1)**

(Total 4 marks)

3.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$ **(2)**

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(b) Find **B**, giving your answer in its simplest form. **(3)**

(Total 5 marks)

4. Two transformations, $\mathrm{T_A}$ and $\mathrm{T_B}$, are represented by matrices $\mathbf{A}$ and $\mathbf{B}$ respectively.

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(a) **(i)** Describe the transformation $\mathrm{T_A}$. **[1]**

(ii) Explain geometrically why $\mathbf{A}^{-1} = \mathbf{A}$. **[1]**

The matrix $\mathbf{B}$ is given by $\mathbf{B} = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$.

(b) Describe the transformation $\mathrm{T_B}$. **[2]**

The transformation $\mathrm{T_C}$ is equivalent to $\mathrm{T_A}$ followed by $\mathrm{T_B}$.

(c) Determine the single matrix which represents $\mathrm{T_C}$. **[2]**

(Total 6 marks)

5.

$$\mathbf{M} = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}$$

(a) Find the value of $\det \mathbf{M}$ **(1)**

The triangle T has vertices at the points $(4, 1)$, $(6, k)$ and $(12, 1)$, where k is a constant.

The triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 216 square units,

(b) find the possible values of k. **(5)**

(Total 6 marks)