QQQ – Core Pure 1 – Chapter 7 – Linear Transformations (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

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1. Find two invariant points under the transformation given by $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ **[2 marks]**

(Total 2 marks)

2.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Describe fully the single geometric transformation A represented by the matrix $\mathbf{A}$. **(2)**

$$\mathbf{B} = \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

The transformation B is represented by the matrix $\mathbf{B}$.

The transformation A followed by the transformation B is the transformation C, which is represented by the matrix $\mathbf{C}$.

To determine matrix $\mathbf{C}$, a student attempts the following matrix multiplication.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

(b) State the error made by the student. **(1)**

(c) Determine the correct matrix $\mathbf{C}$. **(1)**

(Total 4 marks)

3.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$ **(2)**

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(b) Find **B**, giving your answer in its simplest form. **(3)**

(Total 5 marks)

4. Two transformations, $\mathrm{T_A}$ and $\mathrm{T_B}$, are represented by matrices $\mathbf{A}$ and $\mathbf{B}$ respectively.

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(a) **(i)** Describe the transformation $\mathrm{T_A}$. **[1]**

(ii) Explain geometrically why $\mathbf{A}^{-1} = \mathbf{A}$. **[1]**

The matrix $\mathbf{B}$ is given by $\mathbf{B} = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$.

(b) Describe the transformation $\mathrm{T_B}$. **[2]**

The transformation $\mathrm{T_C}$ is equivalent to $\mathrm{T_A}$ followed by $\mathrm{T_B}$.

(c) Determine the single matrix which represents $\mathrm{T_C}$. **[2]**

(Total 6 marks)

5.

$$\mathbf{M} = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}$$

(a) Find the value of $\det \mathbf{M}$ **(1)**

The triangle T has vertices at the points $(4, 1)$, $(6, k)$ and $(12, 1)$, where k is a constant.

The triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 216 square units,

(b) find the possible values of k. **(5)**

(Total 6 marks)

Solutions

1. AQA AS June 2018 Paper 1 Q7

Scheme	Marks	AO
Obtains two equations in x and y. May be seen as a single vector equation. At least one equation must be correct. Accept a pair of letters other than x and y. Ignore any subsequent incorrect working.	M1	1.1a
Obtains any two correct invariant points, with no incorrect points. Condone correct points given as position vectors. NMS: Correct answer scores 2/2. NMS: One correct invariant point and only one incorrect point scores SC1.	A1	1.1b
	(2 marks)	

Typical solution

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2x + 3y \\ x + 4y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + 3y = x \quad \text{and} \quad x + 4y = y$$

$$x = -3y$$

$\therefore$ two invariant points are $(0, 0)$ and $(-3, 1)$

2. Edexcel AS June 2023 Paper 1 Q3

Mark scheme (a)

Scheme	Marks	AO
Rotation	B1	1.1b
30 degrees or $\frac{\pi}{6}$ about the x-axis Ignore any reference to direction	B1	1.1b

	(2)	

Notes

B1: Identifies the single transformation as a rotation only

B1: Correct angle and axis. Ignore any reference to direction.

Note x-plane, zy-plane and $x = 0$ are 2nd B0

Any additional incorrect statements is 2nd B0

Mark scheme (b)

Scheme	Marks	AO
They have found **AB** when they should find **BA** Multiplication is the wrong way round It should be **BA** Matrix B should be on the left instead of the right Student has done transformation B followed by transformation A It should be $\begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$	B1	2.3
	(1)	

Notes

B1: Explains that they should be multiplied the other way around

Mark scheme (c)

Scheme	Marks	AO
$\left\{ \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \right\} \begin{pmatrix} 1 & \frac{3\sqrt{3}}{2} & -\frac{3}{2} \\ \sqrt{3} & \frac{5\sqrt{3}}{2} & \frac{15}{2} \\ 1 & \sqrt{3} & -1 \end{pmatrix}$	B1	1.1b

$\left\{ \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \right\} \begin{pmatrix} 1 & \frac{3\sqrt{3}}{2} & -1.5 \\ \sqrt{3} & \frac{5\sqrt{3}}{2} & 7.5 \\ 1 & \sqrt{3} & -1 \end{pmatrix}$		
	(1)	
	(4 marks)	

Notes

B1: Correct exact matrix

Note: $5\sqrt{3} \times \frac{\sqrt{3}}{2}$ must be simplified to $\frac{15}{2}$

Condone $\frac{2\sqrt{3}}{2}$ not simplified

3. Edexcel FP1 June 2017 Q2

Mark scheme (a)

Scheme	Marks
$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$, $\mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$	
$\mathbf{A}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$	M1 A1
	(2)

Notes

M1: Either $\frac{1}{10}$ or $\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$

A1: Correct matrix seen.

Mark scheme (b)

Scheme	Marks
Way 1 $\mathbf{P} = \mathbf{AB}$ $\Rightarrow \mathbf{A}^{-1}\mathbf{P} = \mathbf{A}^{-1}\mathbf{AB} \Rightarrow \mathbf{B} = \mathbf{A}^{-1}\mathbf{P}$	

$\mathbf{B} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$	M1
$= \begin{pmatrix} 2 & 1 \\ 1 & -4 \end{pmatrix}$	A1 A1
	(3)
	(5 marks)

Notes

M1: Multiplies **their** $\mathbf{A}^{-1}$ by **P** in correct order. This substituted statement is sufficient.

A1: At least 2 elements correct or $k\begin{pmatrix} 20 & 10 \\ 10 & -40 \end{pmatrix}$ oe. May be unsimplified

A1: Correct simplified matrix.

(b) Way 2

Scheme	Marks
$\{\mathbf{P} = \mathbf{AB} \Rightarrow\}$ $\begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$	
$\begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix} = \begin{pmatrix} 2a-c & 2b-d \\ 4a+3c & 4b+3d \end{pmatrix}$	M1
$\Rightarrow a = 2,\ c = 1,\ b = 1,\ d = -4$	
So, $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 1 & -4 \end{pmatrix}$	A1 A1
	(3)

M1: Attempt to multiply **A** by **B** in the correct order and puts equal to **P**

A1: At least 2 elements are correct.

A1: Correct matrix.

4. OCR A AS June 2025 Paper 1 Q4

Mark scheme (a)

Scheme	Marks	AO
(i) Reflection in the line $y = x$.	B1	1.2
	[1]	

(ii) If you carry out the same reflection twice you get back to where you started.	B1	2.4
	[1]	

Notes

(a)(i)

B1: Not "mirrored"

(a)(ii)

B1: The idea that the second identical reflection undoes the first.
BOD reference to mirror here
Must be a geometrical argument
B0 if only argument is det(M) = -1

Mark scheme (b)

Scheme	Marks	AO
Rotation	M1	2.2a
By $(+)\frac{1}{3}\pi$ (radians) anticlockwise about O.	A1	1.1
	[2]	

Notes

M1: Recognising the matrix as a rotation.

A1: Angle and sense given.
Allow 60° for angle
Allow omissions of O
A0 if includes another transformation as well

Mark scheme (c)

Scheme	Marks	AO
T_C is represented by $\mathbf{BA}$	M1	2.2a
$= \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix}$	A1	1.1
	[2]	

Notes

M1: Recognising the correct order for the multiplication

Determine: Needs some evidence. Could see the correct order of matrices before multiplication

A1: If **M0** then **SCB1** for correct answer with no working.

5. Edexcel FP1 June 2018 Q6

Mark scheme (a)

Scheme	Marks
$\{\det \mathbf{M} = (8)(2) - (-1)(-4)\} \Rightarrow \det \mathbf{M} = 12$	B1
	(1)

Notes

B1: 12

Mark scheme (b)

Scheme	Marks
Area $T = \dfrac{216}{12}$ $\{= 18\}$	M1
$h = \pm(1-k)$	M1
$\frac{1}{2}8(k-1) = 18$ or $\frac{1}{2}8(1-k) = 18$ or $(k-1) = \dfrac{18}{4}$ or $(1-k) = \dfrac{18}{4}$ or $\left\{\frac{1}{2}8h = 18\right\} \Rightarrow h = \dfrac{9}{2},\ k = 1 \pm \dfrac{9}{2}$	ddM1
$\Rightarrow k = 5.5$ or $k = -3.5$	A1 A1
	(5)
	(6 marks)

Notes

M1: Area $T = \dfrac{216}{\text{their ” det } \mathbf{M}\text{”}}$

M1: Uses $(k-1)$ or $(1-k)$ in their solution.

ddM1: dependent on the two previous M marks

$\frac{1}{2}8(k-1)$ or $\frac{1}{2}8(1-k) = \dfrac{216}{\text{their ” det } \mathbf{M}\text{”}}$

or $(k-1)$ or $(1-k) = \dfrac{216}{4(\text{their }''\det \mathbf{M}'')}$

or $h = \dfrac{216}{4(\text{their }''\det \mathbf{M}'')}$, $k = 1 \pm \dfrac{216}{4(\text{their }''\det \mathbf{M}'')}$

A1: At least one of either $k = 5.5$ or $k = -3.5$

A1: Both $k = 5.5$ and $k = -3.5$

ALT (b)

Scheme	Marks
$\mathbf{T}' = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 4 & 6 & 12 \\ 1 & k & 1 \end{pmatrix}$	
$\mathbf{T}' = \begin{pmatrix} 31 & 48-k & 95 \\ -14 & -24+2k & -46 \end{pmatrix}$ or 18 seen	M1
$\dfrac{1}{2}\begin{vmatrix} 31 & 48-k & 95 & 31 \\ -14 & -24+2k & -46 & -14 \end{vmatrix} = 216$ **or** $\dfrac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	M1
$\dfrac{1}{2}\lvert -744+62k+672-14k-2208+46k+2280-190k-1330+1426\rvert = 216$ $\dfrac{1}{2}\lvert 4k-6+6-12k+12-4\rvert = 18$	ddM1
$\dfrac{1}{2}\lvert 96-96k\rvert = 216$ or $\dfrac{1}{2}\lvert 8-8k\rvert = 18$	
So, $1-k = 4.5$ or $k-1 = 4.5$	
$\Rightarrow k = -3.5$ or $k = 5.5$	A1 A1
	(5)

M1: At least 5 out of 6 elements are correct or 18 seen..

M1: $\dfrac{1}{2}\lvert \text{their } \mathbf{T}'\rvert = 216$ or $\dfrac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$

ddM1: Dependent on the two previous M marks. Full method of evaluating a determinant.

A1: At least one of either $k = -3.5$ or $k = 5.5$

A1: Both $k = -3.5$ and $k = 5.5$