

QQQ – Core Pure 1 – Chapter 6 – Matrices (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The matrices **A**, **B** and **C** are defined as follows:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & 3 \end{pmatrix}.$$

Calculate all possible products formed from two of these three matrices.

[4]

(Total 4 marks)

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2.

$$\mathbf{M} = \begin{pmatrix} a & 2 & -3 \\ 2 & 3 & 0 \\ 4 & a & 2 \end{pmatrix} \quad \text{where } a \text{ is a constant}$$

(a) Show that **M** is non-singular for all values of a .

(2)

(b) Determine, in terms of a , $\mathbf{M}^{-1}$

(4)

(Total 6 marks)

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3. Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & -3 \\ -2 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & -5 \\ 0 & 1 \end{pmatrix}$.

(a) Find $2\mathbf{A} - 4\mathbf{B}$.

[2]

(b) Write down the matrix **C** such that $\mathbf{AC} = 2\mathbf{A}$.

[1]

(c) Find the value of $\det \mathbf{A}$.

[1]

(d) **In this question you must show detailed reasoning.**

Use $\mathbf{A}^{-1}$ to solve the equations $4x - 3y = 7$ and $-2x + 2y = 9$.

[3]

(Total 7 marks)

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4. Three planes have equations

$$kx + y - 2z = 0$$

$$2x + 3y - 6z = -5$$

$$3x - 2y + 5z = 1$$

where k is a constant.

Investigate the arrangement of the planes for each of the following cases. If in either case the planes meet at a unique point, find the coordinates of that point.

(a) $k = -1$ **[3]**

(b) $k = \frac{2}{3}$ **[4]**

(Total 7 marks)

Solutions

1.

Scheme	Marks	AO
$\mathbf{BA} = \begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 8 \end{pmatrix}$	M1 A1	1.1a 1.1
$\mathbf{AC} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1 \ 3) = \begin{pmatrix} 1 & 3 \\ 2 & 6 \\ 3 & 9 \end{pmatrix}$	A1	1.1
$\mathbf{CB} = (1 \ 3) \begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix} = (5 \ -3 \ 12)$	A1	1.1
	[4]	

Notes

M1: **BA**, **AC** or **CB** calculated with correct shape

allow one arith error for M1

A1: (3rd) deduct A1 for any incorrect products pursued

2.

Mark scheme (a)

Scheme	Marks	AO
$\det(\mathbf{M}) = a(6) - 2(4) - 3(2a - 12)$	M1	1.1b
$\det(\mathbf{M}) = 28 \neq 0$ therefore, non-singular for all values of a	A1	2.4
	(2)	

Notes

M1: Finds the determinant of the matrix **M**. Must be seen in part (a). Allow one slip if no method shown.

A1: Correct value for determinant, states doesn't equal 0 (accept > 0) and draws the conclusion that the matrix is non-singular. If non-singular meaning determinant is non-zero is given in a preamble then accept a minimal conclusion (e.g. "hence shown"), but there must be a conclusion.

Mark scheme (b)

Scheme	Marks	AO
Finds the matrix of minors $\begin{pmatrix} 6 & 4 & 2a - 12 \\ 4 + 3a & 2a + 12 & a^2 - 8 \\ 9 & 6 & 3a - 4 \end{pmatrix}$	M1	1.1b
Finds the matrix of cofactors and transposes. $\begin{pmatrix} 6 & -4 - 3a & 9 \\ -4 & 2a + 12 & -6 \\ 2a - 12 & 8 - a^2 & 3a - 4 \end{pmatrix}$	M1	1.1b
$\frac{1}{28} \begin{pmatrix} 6 & -4 - 3a & 9 \\ -4 & 2a + 12 & -6 \\ 2a - 12 & 8 - a^2 & 3a - 4 \end{pmatrix}$	M1 A1	1.1b 2.1
	(4)	
	(6 marks)	

Notes

M1: Finds the matrix of minors, at least 5 correct values.

M1: Finds the matrix of cofactors and transposes (in either order). Note: some will do all these steps in one go, which is fine as long as it is clear what they have done. Allow minor slips if the process is clearly correct.

M1: Completes the process to find the inverse matrix, divides by the determinant.

A1: Correct matrix.

3.

Mark scheme (a)

Scheme	Marks	AO
$\mathbf{C} = 2 \begin{pmatrix} 4 & -3 \\ -2 & 2 \end{pmatrix} - 4 \begin{pmatrix} 3 & -5 \\ 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} 8 & -6 \\ -4 & 4 \end{pmatrix} - \begin{pmatrix} 12 & -20 \\ 0 & 4 \end{pmatrix}$ $= \begin{pmatrix} 8 - 12 & -6 - -20 \\ -4 - 0 & 4 - 4 \end{pmatrix}$	M1	1.1
$= \begin{pmatrix} -4 & 14 \\ -4 & 0 \end{pmatrix}$	A1	1.1

	[2]	
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Notes

M1: Sufficient working to demonstrate knowledge of scalar multiplication of a matrix and subtraction of matrices. Can be implied by 3 out of 4 entries correct.

A1: Or BC

Mark scheme (b)

Scheme	Marks	AO
$(\mathbf{C} =) \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \text{ or } 2\mathbf{I}$	B1	2.2a
	[1]	

Mark scheme (c)

Scheme	Marks	AO
$(\det \mathbf{A} = 4 \times 2 - (-3)(-2) = 8 - 6 =) 2$	B1	1.1
	[1]	

Mark scheme (d)

Scheme	Marks	AO
$\begin{pmatrix} 4 & -3 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$	M1	1.1
$\frac{1}{2} \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 7 \\ 9 \end{pmatrix} \text{ or } \mathbf{A}^{-1} \begin{pmatrix} 7 \\ 9 \end{pmatrix} \text{ if } \mathbf{A}^{-1} \text{ defined}$	M1	1.1
so $x = \frac{41}{2}, y = 25$	A1	1.1
	[3]	

Notes

M1: DR Expressing the system in matrix form. Can be implied by the next line. Matrix method must be used. Any other method 0/3.

M1: Forming correct solution as matrix/vector product with inverse matrix. FT their $\det \mathbf{A}$ from (c). Except for this, inverse must be correct.

A1: $20\frac{1}{2}$ or 20.5

Condone $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{41}{2} \\ 25 \end{pmatrix}$ but not $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 41 \\ 50 \end{pmatrix}$

4.

Mark scheme (a)

Scheme	Marks	AO
when $k = -1$, $\det \mathbf{M} \neq 0$ [so meet at a point]	M1	1.1
$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} 0 \\ -5 \\ 1 \end{pmatrix}$	M1	1.1
by calculator, point of intersection is $(-1, 3, 2)$	A1	1.1
	[3]	

Notes

M1: (1st) calculating determinant
or finding $\mathbf{M}^{-1}$

M1: (2nd) solving (soi)
or by sim equations

A1: BC, allow $\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$

If correct ans found by solving simultaneously SCB3

Mark scheme (b)

Scheme	Marks	AO
when $k = 2/3$, $\det \mathbf{M} = 0$,	M1	3.1a
so no unique point of intersection	A1	1.1
[coeffts of 2 nd plane are 3 times those of first]		3.2a
so first two planes are parallel	B1	
and intersected by third plane	B1	3.2a
	[4]	

Notes

M1: or 2 planes are parallel