

QQQ – Core Pure 1 – Chapter 6 – Matrices (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 0 & 5 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 4 \\ 7 & 1 \end{bmatrix}$$

Calculate **AB**

Circle your answer.

[1 mark]

- $\begin{bmatrix} 3 & 5 \\ 7 & 6 \end{bmatrix}$
- $\begin{bmatrix} 0 & 20 \\ 21 & 12 \end{bmatrix}$
- $\begin{bmatrix} 0 & 4 \\ 0 & 5 \end{bmatrix}$
- $\begin{bmatrix} 7 & 13 \\ 35 & 5 \end{bmatrix}$

(Total 1 marks)

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2. Given that k is a real number and that

$$\mathbf{A} = \begin{pmatrix} 1+k & k \\ k & 1-k \end{pmatrix}$$

find the exact values of k for which **A** is a singular matrix. Give your answers in their simplest form.

(3)

(Total 3 marks)

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3.

$$\begin{pmatrix} x & 9 \\ y & z \end{pmatrix} - 3 \begin{pmatrix} z & y \\ z & y \end{pmatrix} = k\mathbf{I}$$

where x, y, z and k are constants.

Determine the value of x , the value of y and the value of z .

(4)

(Total 4 marks)

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4. The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix}$$

(a) Find $\det \mathbf{A}$

[1 mark]

(b) Find $\mathbf{A}^{-1}$

[1 mark]

(c) Given that $\mathbf{AB} = \begin{bmatrix} 9 & 6 \\ 5 & 12 \end{bmatrix}$ and $\mathbf{M} = 2\mathbf{A} + \mathbf{B}$ find the matrix $\mathbf{M}$

[3 marks]

(Total 5 marks)

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5.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -3 \\ 4 & -2 & 1 \\ 3 & 5 & -2 \end{pmatrix}$$

(a) Find $\mathbf{M}^{-1}$ giving each element in exact form.

(2)

(b) Solve the simultaneous equations

$$2x + y - 3z = -4$$

$$4x - 2y + z = 9$$

$$3x + 5y - 2z = 5$$

(2)

(c) Interpret the answer to part (b) geometrically.

(1)

(Total 5 marks)

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6. The equations of three planes are

$$\begin{aligned}2x + y + 3z &= 3, \\3x - y - 2z &= 2, \\-4x + 3y + 7z &= k,\end{aligned}$$

where k is a constant.

- (a) By considering a suitable determinant, show that the planes do **not** meet at a single point. [2]
- (b) Given that the planes form a sheaf, determine the value of k . [4]

(Total 6 marks)

Solutions

1.

Scheme	Marks	AO
Circles the correct answer	B1	1.1b
	(1 mark)	

Typical solution

$$\begin{bmatrix} 7 & 13 \\ 35 & 5 \end{bmatrix}$$

2.

Scheme	Marks
Determinant of $\mathbf{A} = (1 - k)(1 + k) - k^2 = 0$	M1
$1 - k + k - k^2 - k^2 (= 0)$ $1 - 2k^2 (= 0)$	A1
So $k = \frac{\pm\sqrt{2}}{2}$	A1
	(3)
	(3 marks)

Notes

M1: for attempting $ad - bc = 0$ with ' $= 0$ ' seen or **used** later in the solution.

A1: Correct (unsimplified) expression on LHS or correct equation after brackets expanded.

A1: Accept $\pm\frac{\sqrt{2}}{2}$, $\pm\frac{1}{\sqrt{2}}$, $\pm\sqrt{\frac{1}{2}}$, $\pm\sqrt{0.5}$. Must have $\pm$ for mark.

3.

Scheme	Marks	AO
$y = 3$	B1	2.2a
$z = \frac{\text{their } y}{3} = \dots \{1\}$	Blft	1.1b

Uses $z - 3y = k \Rightarrow k = -8$ and $x - 3z = k \Rightarrow x = k + 3z = \text{their } k + 3 \times \text{their } z$ leading to a value for x Alternatively uses $x - 3z = k = z - 3y$ with values for y and z to find a value for x .	M1	3.1a
$x = -5$	A1	1.1b
	(4)	
	(4 marks)	

Notes

B1: $y = 3$

B1ft: Follow through on the value of z which comes from their y divided by 3

M1: A complete method to find the value of x . Uses $z - 3y = k$ to find a value for k then finds a value for x using $x - 3z = k$ and their values for z and k . Condone a slip with the coefficients if the intention is clear but must have the correct letters.

Alternatively uses $x - 3z = k = z - 3y$ with values for y and z to find a value for x .

A1: $x = -5$

Correct answers only scores full marks.

4.

Mark scheme (a)

Scheme	Marks	AO
Obtains the correct determinant.	B1	1.1b
	(1)	

Typical solution

$$\begin{aligned}\det \mathbf{A} &= 5 \times 4 - (-3) \times 2 \\ &= 26\end{aligned}$$

Mark scheme (b)

Scheme	Marks	AO
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Obtains the correct inverse matrix ACF FT their determinant	B1F	1.1b
	(1)	

Typical solution

$$\mathbf{A}^{-1} = \frac{1}{26} \begin{bmatrix} 4 & -2 \\ 3 & 5 \end{bmatrix}$$

Mark scheme (c)

Scheme	Marks	AO
Selects a method to find either matrix B or matrix AM e.g. calculates $\mathbf{A}^{-1}\mathbf{AB}$ with their $\mathbf{A}^{-1}$ or calculates $2\mathbf{A}^2 + \mathbf{AB}$ or writes four simultaneous equations in w, x, y, z where $\begin{bmatrix} 9 & 6 \\ 5 & 12 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix}$	M1	3.1a
Obtains a correct matrix for B or AM i.e. $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ or $\begin{bmatrix} 47 & 42 \\ -49 & 32 \end{bmatrix}$ PI by a correct matrix M FT their $\mathbf{A}^{-1}$	A1F	1.1b
Obtains matrix M FT their $\mathbf{A}^{-1}$	A1F	2.2a
	(3)	
	(5 marks)	

Typical solution

$$\mathbf{B} = \mathbf{A}^{-1}\mathbf{AB}$$

$$\begin{aligned}\mathbf{B} &= \frac{1}{26} \begin{bmatrix} 4 & -2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 9 & 6 \\ 5 & 12 \end{bmatrix} \\ &= \frac{1}{26} \begin{bmatrix} 26 & 0 \\ 52 & 78 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}\mathbf{M} &= 2 \begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 11 & 4 \\ -4 & 11 \end{bmatrix}\end{aligned}$$

5.

Mark scheme (a)

Scheme	Marks	AO
$\mathbf{M}^{-1} = \frac{1}{69} \begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}$	B1	1.1b
	B1	1.1b
	(2)	

Notes

B1: Evidence that the determinant is ± 69 (may be implied by their matrix e.g. where entries are not in

exact form: $\pm \begin{pmatrix} 0.014 & 0.188 & 0.072 \\ -0.159 & -0.072 & 0.203 \\ -0.377 & 0.101 & 0.116 \end{pmatrix}$) (Should be mostly correct)

Must be seen in part (a).

B1: Fully correct inverse with all elements in **exact** form

Mark scheme (b)

Scheme	Marks	AO
$\frac{1}{69} \begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix} \begin{pmatrix} -4 \\ 9 \\ 5 \end{pmatrix} = \dots$	M1	1.1b
$x = 2, y = 1, z = 3$ or $(2, 1, 3)$ or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$	A1	1.1b

	(2)	
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Notes

M1: Any complete method to find the values of x , y and z (Must be using **their inverse** if using the method in the main scheme)

A1: Correct coordinates

A solution not using the inverse requires a complete method to find values for x , y and z for the method mark.

Correct coordinates only scores both marks.

Mark scheme (c)

Scheme	Marks	AO
The point where three planes meet	B1ft	2.2a
	(1)	
	(5 marks)	

Notes

B1: Describes the correct geometrical configuration.

Must include the two ideas of **planes** and **meet in a point** with no contradictory statements.

This is dependent on having obtained a unique point in part (b)

6.

Mark scheme (a)

Scheme	Marks	AO
$\begin{vmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ -4 & 3 & 7 \end{vmatrix} [= 0]$	M1	1.1a
$\det = 0 \Rightarrow$ planes do not meet at a point	A1	1.1
	[2]	

Notes

M1: finding determinant of correct matrix

Mark scheme (b)

Scheme	Marks	AO
Method 1: use 2 equations to find e.g. z and y in terms of x, and substitute in 3rd equation e.g. $(1) + (2): 5x + z = 5,$	M1	2.1
$\Rightarrow z = 5 - 5x, y = 13x - 12$	M1	2.1
substitute in (3): $-4x + 39x - 36 + 35 - 35x = k$	M1	2.1
$\Rightarrow k = -1$	A1	2.2a
	[4]	

Notes

M1: (1st) eliminating one variable

M1: (2nd) finding e.g. y and z in terms of x

M1: (3rd) substituting in 3rd equation

Method 2: eliminate one variable from 2 pairs of eqns, and compare eqns in 2 remaining v'bles, e.g.

Scheme	Marks
from (1) and (2): $5x + z = 5$ (4)	M1
from (1) and (3): $10x + 2z = 9 - k$ (5)	M1
$\Rightarrow 9 - k = 10$	M1
$\Rightarrow k = -1$	A1

M1: (1st) eliminating one variable using two equations

M1: (2nd) eliminating same variable using two other equations

M1: (3rd) eliminating both v'bles using (4) and (5) equations (coeffs in (4) and (5) must be correct)

Method 3: find linear combination by inspection

Scheme	Marks
$(3) = (1) - 2 \times (2)$	M3
$\Rightarrow k = 3 - 2 \times 2 = -1$	A1

M3: oe

A1: for $k = -1$ unsupported, allow SCB3

Method 4: substitute a value of x , y or z , e.g.

Scheme	Marks
substituting $x = 0$ $y + 3z = 3$, $-y - 2z = 2$	M1
$\Rightarrow z = 5$, $y = -12$	A1
substitute in (3): $-36 + 35 = k$	M1
$\Rightarrow k = -1$	A1

M1: (1st) any value for x , y or z substituted

A1: (1st) solve 2 equations for y , z

M1: (2nd) substitute values into third equation