

QQQ – Core Pure 1 – Chapter 5 – Volumes of Revolution (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1.

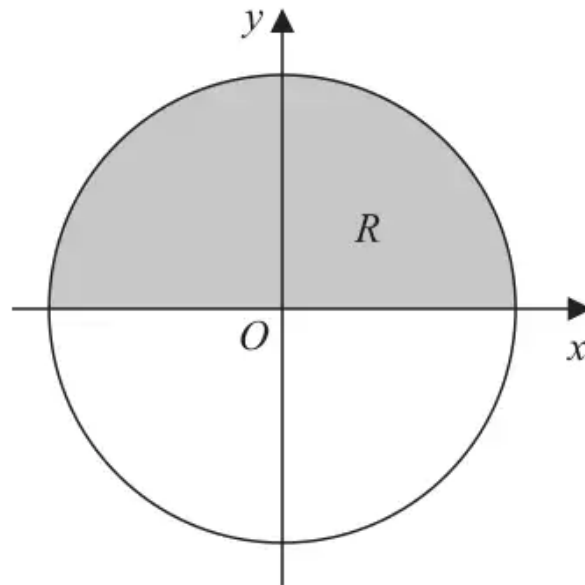


Figure 1

Figure 1 shows a circle with radius r and centre at the origin.

The region R , shown shaded in Figure 1, is bounded by the x -axis and the part of the circle for which $y > 0$

The region R is rotated through 360° about the x -axis to create a sphere with volume V

Use integration to show that $V = \frac{4}{3}\pi r^3$ (5)

(Total 5 marks)

2.

(a) Find

$$\int (2x - 1)^{\frac{3}{2}} dx$$

giving your answer in its simplest form.

(2)

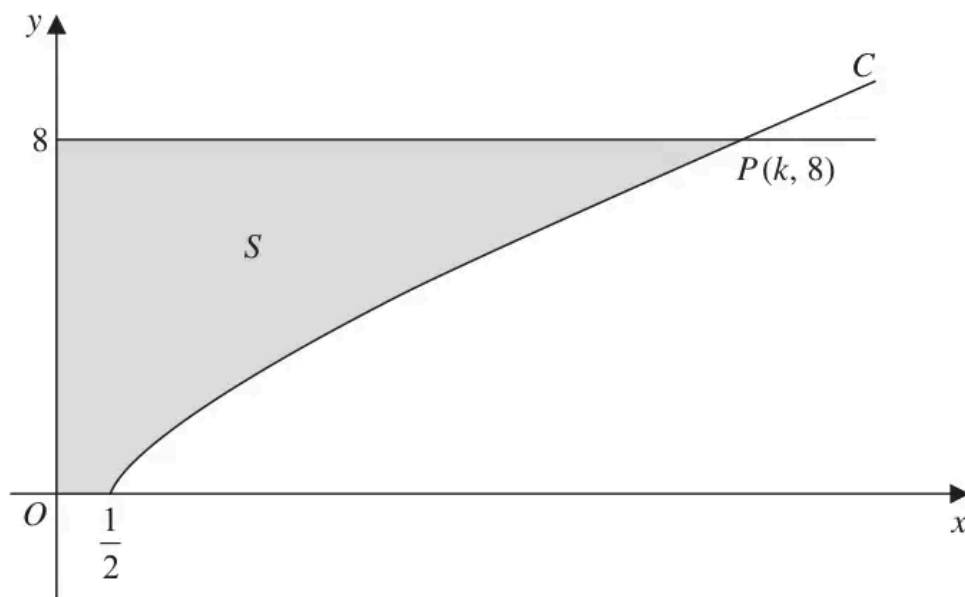


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = (2x - 1)^{\frac{3}{4}}, \quad x \geq \frac{1}{2}$$

The curve C cuts the line $y = 8$ at the point P with coordinates $(k, 8)$, where k is a constant.

(b) Find the value of k .

(2)

The finite region S , shown shaded in Figure 3, is bounded by the curve C , the x -axis, the y -axis and the line $y = 8$. This region is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Find the exact value of the volume of the solid generated.

(4)

(Total 8 marks)

3.

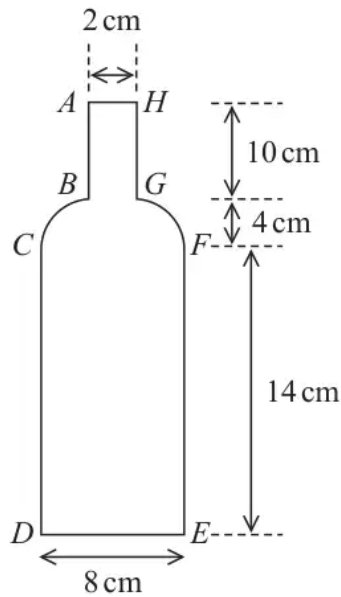


Figure 1

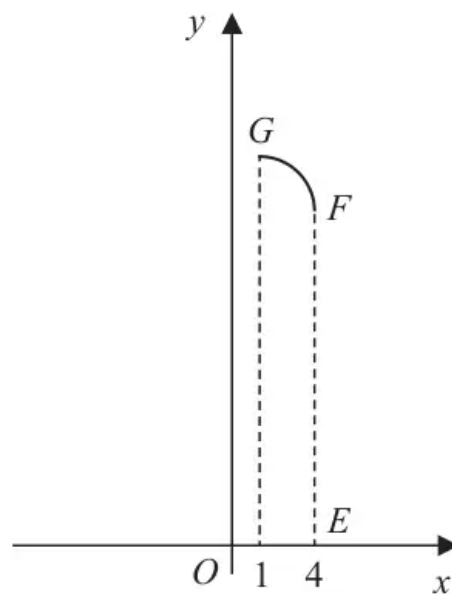


Figure 2

A mathematics student is modelling the profile of a glass bottle of water. Figure 1 shows a sketch of a central vertical cross-section $ABCDEFGH$ of the bottle with the measurements taken by the student.

The horizontal cross-section between CF and DE is a circle of diameter 8 cm and the horizontal cross-section between BG and AH is a circle of diameter 2 cm.

The student thinks that the curve GF could be modelled as a curve with equation

$$y = ax^2 + b \quad 1 \leq x \leq 4$$

where a and b are constants and O is the fixed origin, as shown in Figure 2.

- (a) Find the value of a and the value of b according to the model. (2)
- (b) Use the model to find the volume of water that the bottle can contain. (7)
- (c) State a limitation of the model. (1)

The label on the bottle states that the bottle holds approximately 750 cm^3 of water.

- (d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning. (1)

(Total 11 marks)