

QQQ – Core Pure 1 – Chapter 5 – Volumes of Revolution (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The function f is defined by

$$f(x) = 2x + 3 \quad 0 \leq x \leq 5$$

The region R is enclosed by $y = f(x)$, $x = 5$, the x -axis and the y -axis.

The region R is rotated through 2π radians about the x -axis.

Give an expression for the volume of the solid formed.

Tick (✓) **one** box.

[1 mark]

- $\pi \int_0^5 (2x + 3) \, dx$
- $\pi \int_0^5 (2x + 3)^2 \, dx$
- $2\pi \int_0^5 (2x + 3) \, dx$
- $2\pi \int_0^5 (2x + 3)^2 \, dx$

(Total 1 marks)

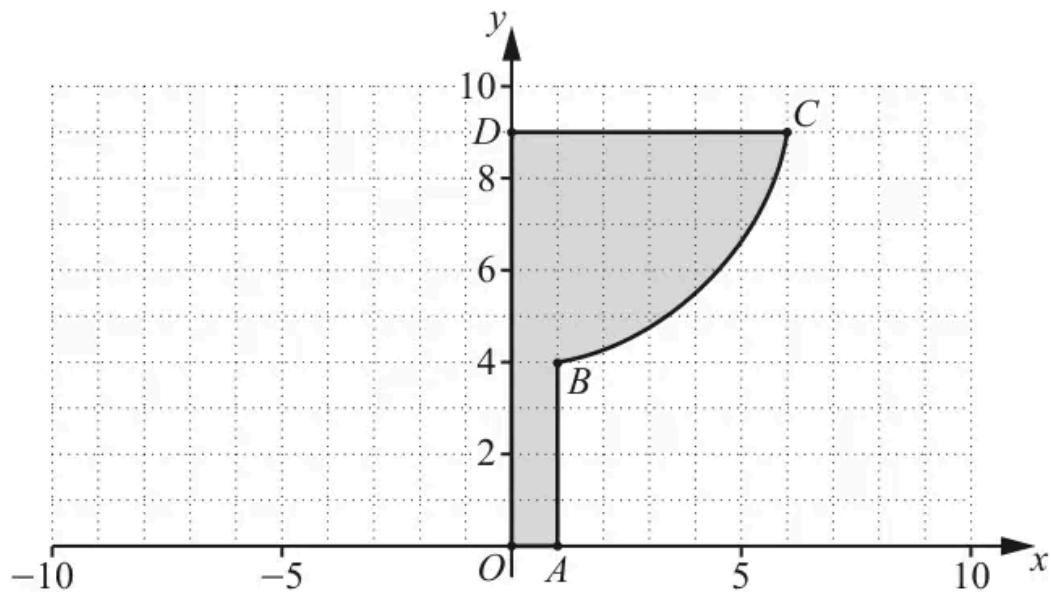
2. O is the origin of a coordinate system whose units are cm.

The points A , B , C and D have coordinates $(1, 0)$, $(1, 4)$, $(6, 9)$ and $(0, 9)$ respectively.

The arc BC is part of the curve with equation $x^2 + (y - 10)^2 = 37$.

The closed shape $OABCD$ is formed, in turn, from the line segments OA and AB , the arc BC and the line segments CD and DO (see diagram).

A funnel can be modelled by rotating $OABCD$ by 2π radians about the y -axis.



Find the volume of the funnel according to the model.

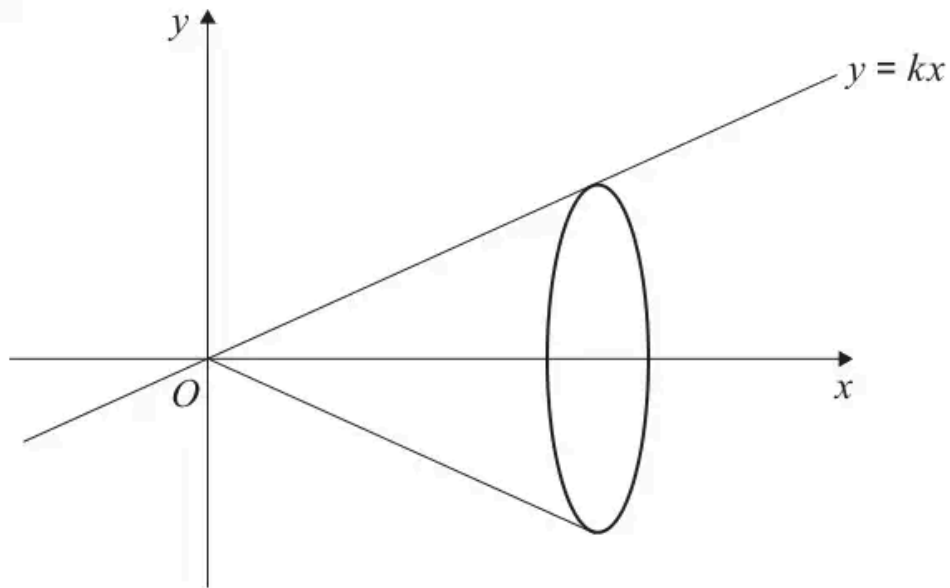
[3]

(Total 3 marks)

3. A segment of the line $y = kx$ is rotated about the x -axis to generate a cone with vertex O .

The distance of O from the centre of the base of the cone is h .

The radius of the base of the cone is r .



(a) Find k in terms of r and h .

[1 mark]

(b) Use calculus to prove that the volume of the cone is

$$\frac{1}{3}\pi r^2 h$$

[3 marks]

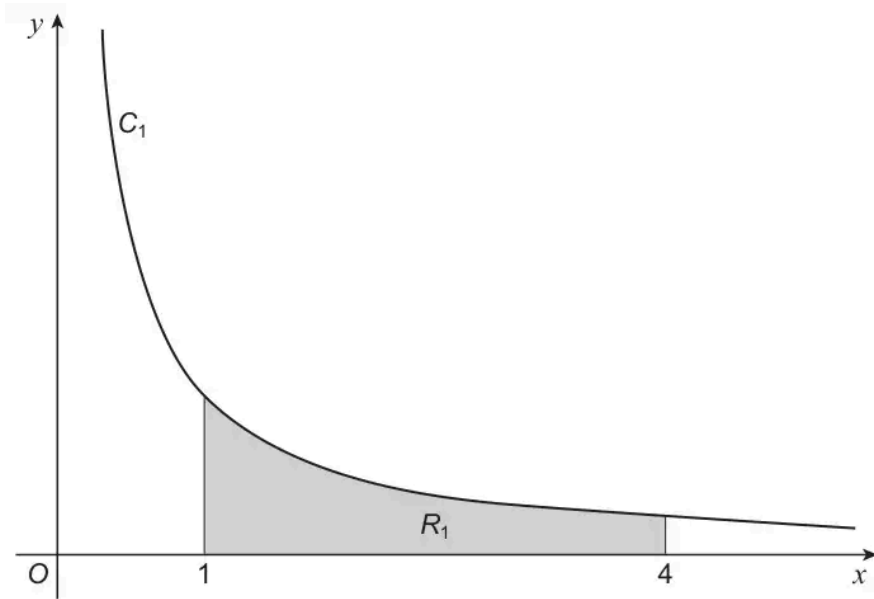
(Total 4 marks)

4. The curve C_1 is given by the equation

$$xy = m$$

where m is a positive constant.

The region R_1 is enclosed by curve C_1 , the x -axis and the lines $x = 1$ and $x = 4$



The region R_1 is rotated through 2π radians about the x -axis.

The volume of the solid generated is V

(a) Find a simplified expression for V in terms of m **[2 marks]**

(b) The curve C_2 is a stretch of C_1 by a factor 5 in the y -direction.

The region R_2 is enclosed by C_2 , the x -axis and the lines $x = 1$ and $x = 4$

The region R_2 is rotated through 2π radians about the x -axis.

The volume of the solid generated by rotating R_2 is 6π

Calculate the value of m **[3 marks]**

(Total 5 marks)

5.

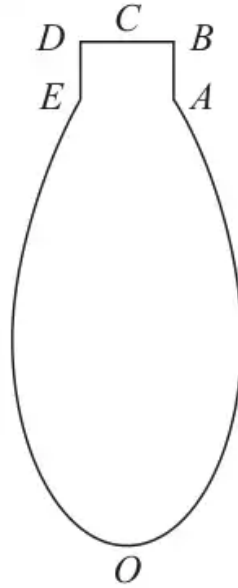


Figure 1

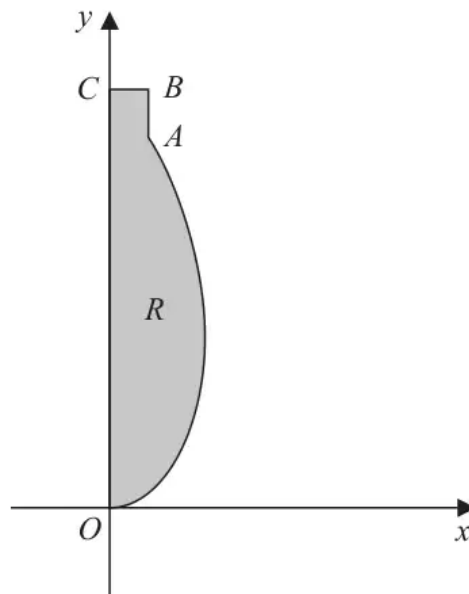


Figure 2

Figure 1 shows the central vertical cross-section, $OABCDEO$, of the design for a solid glass ornament.

Figure 2 shows the finite region, R , which is bounded by the y -axis, the horizontal line CB , the vertical line BA , and the curve AO .

The ornament is formed by rotating the region R through 360° about the y -axis.

The curve AO is modelled by the equation

$$x = ky^2 + \sqrt{y} \quad 0 \leq y \leq 4$$

where k is a constant.

The point A has coordinates $(0.4, 4)$ and the point B has coordinates $(0.4, 4.5)$

The units are centimetres.

- (a) Determine the value of k according to this model. (2)
- (b) Use algebraic integration to determine the exact volume of glass that would be required to make the ornament, according to the model. (7)
- (c) State a limitation of the model. (1)

When the ornament was manufactured, 9 cm^3 of glass was required.

- (d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning. (1)

(Total 11 marks)

Solutions

1.

Scheme	Marks	AO
Ticks the 2 nd box.	B1	1.1b
	(1 mark)	

Typical solution

$$\pi \int_0^5 (2x + 3)^2 \, dx$$

2.

Scheme	Marks	AO
For AB , $V = \pi \times 1^2 \times 4 = 12.566 \dots$		
For BC , $V = \int_a^b \pi x^2 \, dy = \pi \int_4^9 (37 - (y - 10)^2) \, dy$	M1	3.3
$= 356.05 \dots$	A1	1.1
$\Rightarrow$ Total $V = 356.05 \dots + 12.566 \dots = 368.61 \dots$ $= 369 \text{ (cm}^3\text{) to 3 sf}$	A1	3.4
	[3]	

Notes

For AB : 4π

M1: Split into two parts and use formulae

An integral and an attempt at the volume of a cylinder must be seen

A1: Integration – ignore limits **BC**

$$\frac{340}{3}\pi$$

A1: Units are not required

$$\frac{352}{3}\pi$$

3.

Mark scheme (a)

Scheme	Marks	AO
Obtains the correct expression $k = \frac{r}{h}$.	B1	1.1b

Typical solution

$$\frac{r}{h}$$

Mark scheme (b)

Scheme	Marks	AO
Uses the formula for volume of revolution $V = \pi \int m x^2 dx$. Condone missing π , dx and missing or incorrect limits.	M1	1.1a
Correctly integrates their $(kx)^2$, with an expression for k in terms of r and h .	M1	1.1a
Completes a rigorous proof to show that $V = \frac{1}{3}\pi r^2 h$.	R1	2.1
	(4 marks)	

Typical solution

$$\begin{aligned}
 \text{Volume} &= \pi \int_0^h \left(\frac{rx}{h}\right)^2 dx \\
 &= \pi \int_0^h \frac{r^2 x^2}{h^2} dx \\
 &= \pi \left[\frac{r^2 x^3}{3h^2} \right]_0^h \\
 &= \pi \left(\frac{r^2 h^3}{3h^2} - 0 \right) = \frac{1}{3}\pi r^2 h
 \end{aligned}$$

4.**Mark scheme (a)**

Scheme	Marks	AO
Writes a correct expression for V in terms of x Condone missing dx	M1	1.2

Obtains $\frac{3}{4}m^2\pi$	A1	1.1b
	(2)	

Typical solution

$$\begin{aligned}
 V &= \pi \int_1^4 \left(\frac{m}{x}\right)^2 dx \\
 &= \pi m^2 \left[-\frac{1}{x}\right]_1^4 \\
 &= \frac{3\pi m^2}{4}
 \end{aligned}$$

Mark scheme (b)

Scheme	Marks	AO
Writes a correct expression for the stretched volume in terms of m (and x) eg $\pi \int_1^4 \left(\frac{5m}{x}\right)^2 dx$ or Writes their part (a) $\times 5^2$	M1	3.1a
Forms a correct equation in terms of m only or Forms the equation their part (a) $\times 5^n = 6\pi$ where $n = 1, 2$ or 3 PI by $m^2 = \frac{8}{5}$ or $m^2 = \frac{8}{125}$ oe	M1	1.1a
Obtains $\frac{2}{5}\sqrt{2}$ oe eg $\frac{\sqrt{8}}{5}$ Negative root does not have to be considered.	A1	1.1b
	(3)	
	(5 marks)	

Typical solution

$$\frac{3\pi m^2}{4} \times 5^2 = 6\pi$$

$$m^2 = \frac{8}{25}$$

$$m > 0 \text{ so } m = \frac{2}{5}\sqrt{2}$$

5.

Mark scheme (a)

Scheme	Marks	AO
$(0.4, 4) \Rightarrow 0.4 = k \times 4^2 + \sqrt{4} \Rightarrow k = \dots$	M1	3.3
$k = -0.1$	A1	1.1b
	(2)	

Notes

M1: Substitutes $(0.4, 4)$ into the equation modelling the curve in an attempt to find the value of k

A1: Infers from the data in the model, the value of k

Mark scheme (b)

Scheme	Marks	AO
Cylinder volume = $\pi \times 0.4^2 \times 0.5 = 0.08\pi = \frac{2}{25}\pi$	B1	3.4
Volume generated by curve = $\pi \int x^2 \, dy$ $\pi \int (\sqrt{y} + ky^2)^2 \{dy\} = \pi \int (\sqrt{y} - 0.1y^2)^2 \{dy\}$	M1	3.1b
$= \{\pi\} \int (y + 2ky^{\frac{5}{2}} + k^2y^4) \{dy\}$ $= \{\pi\} \int (y - 0.2y^{\frac{5}{2}} + 0.01y^4) \{dy\}$	A1ft	1.1b
$= \{\pi\} \int_0^1 (y - 0.2y^{\frac{5}{2}} + 0.01y^4) \{dy\}$ $\Rightarrow \{\pi\} [Ay^2 + By^{\frac{7}{2}} + Cy^5]$ at least one of their terms with the correct power	M1	3.4

$= \{\pi\} \left[\frac{y^2}{2} + \frac{4k}{7}y^{\frac{7}{2}} + \frac{k^2}{5}y^5 \right]$ $= \{\pi\} \left[\frac{y^2}{2} - \frac{2}{35}y^{\frac{7}{2}} + \frac{1}{500}y^5 \right]$	A1ft	1.1b
$V = \pi \left(8 - \frac{256}{35} + \frac{256}{125} \right) - (0) + \frac{2}{25}\pi$ $V = \frac{2392}{875}\pi + \frac{2}{25}\pi$	M1	3.4
$V = \frac{2462\pi}{875} \text{ cm}^3$	A1	2.2b
	(7)	

Notes

B1: Uses the information given in the model to establish the correct volume of the cylinder

M1: Uses the model and applies $\pi \int x^2 \{dy\}$, dy not required and π may appear later in their solution. If they find an expression for x^2 first and then substitutes into the formula score M1 even if an incorrect expansion.

A1ft: Correct expression for the volume generated by the curve with the bracket expanded (follow through their k value), dy not required and π may appear later in their solution. Indices need to be processed for this mark, may be seen later in the solution.

M1: Attempts to integrate with at least one power raised by 1

A1ft: Correct integration (follow through on their expression for x^2 as long as there are 3 terms). Need not be simplified.

M1: Uses the correct limits and finds the sum of the 2 volumes. Must come from an attempt at

$\pi \int_0^4 x^2 \{dy\}$ and an attempt at the volume of the cylinder; condone incorrect formula used as long as it is 3 dimensional not an area.

A1: $\frac{2462\pi}{875}$

Use of calculator scores a maximum of B1M1A0M0A0M1A0 volume = $\pi 2.7337...$

Mark scheme (c)

Scheme	Marks	AO
E.g.	B1	3.5b
The equation of the curve may not be a suitable model		

• The sides of the ornament will not be perfectly smooth • There may be flaws/bubbles within the glass • The corner (ABC) may not be a perfect right angle		
	(1)	

Notes

B1: States an acceptable limitation of the model, which is the curve but accept flaws/bubbles in the glass. Measurements may not be accurate, or anything related to thickness is B0

Mark scheme (d)

Scheme	Marks	AO
Makes an appropriate comment that is consistent with their value for the volume and 9 cm^3. Some evidence of making a comparison and draws a conclusion E.g. a good estimate as 8.84 cm^3 is only 0.16 cm^3 less than 9 cm^3 • A volume between 8.5 and 9.5 is a good model • A volume between 8 and 10 can be either a good or bad model • A volume less than 8 or more than 10 is a bad model, over estimate or underestimate • model volume is less, not enough glass would be ordered so it is a bad model, following a correct answer to (b)	B1ft	3.5a
	(1)	
	(11 marks)	

Notes

B1ft: Compares the actual volume to their answer to part (b) and makes an assessment of the model with a reason. If using a percentage error then they must use 9 as the true volume.