

QQQ – Core Pure 1 – Chapter 4 – Roots of Polynomials (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The roots of the equation $x^2 - px - 6 = 0$ are α and β

Find $\alpha^2 + \beta^2$ in terms of p

Circle your answer.

[1 mark]

- $p^2 - 6$
- $p^2 + 6$
- $p^2 - 12$
- $p^2 + 12$

(Total 1 marks)

AQA A2 June 2022 Paper 2 Q3

2. The roots of the equation

$$5x^3 + 2x^2 - 3x + p = 0$$

are α , β and γ

Given that p is a constant, state the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

Circle your answer.

[1 mark]

- $-\frac{3}{5}$
- $-\frac{2}{5}$
- $\frac{2}{5}$
- $\frac{3}{5}$

(Total 1 marks)

AQA AS June 2023 Paper 1 Q4

3. In this question you must show detailed reasoning.

The equation $x^2 - kx + 2k = 0$, where k is a non-zero constant, has roots α and β .

Find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ in terms of k , simplifying your answer.

[4]

(Total 4 marks)

OCR MEI AS June 2023 Paper 1 Q2

4. The roots of the equation $4x^4 - 2x^3 - 3x + 2 = 0$ are α, β, γ and δ . By using a suitable substitution, find a quartic equation whose roots are $\alpha + 2, \beta + 2, \gamma + 2$ and $\delta + 2$ giving your answer in the form $at^4 + bt^3 + ct^2 + dt + e = 0$, where a, b, c, d , and e are integers.

[5]

(Total 5 marks)

OCR A AS June 2023 Paper 1 Q1

5. In this question you must show detailed reasoning.

The equation $z^3 + 2z^2 + kz + 3 = 0$, where k is a constant, has roots $\alpha, \frac{1}{\alpha}$ and β .

Determine the roots in exact form.

[6]

(Total 6 marks)

OCR MEI AS June 2022 Paper 1 Q4

6. The roots of the equation

$$x^3 - 2x^2 + 4x - 5 = 0$$

are p, q and r .

Without solving the equation, find the value of

(i) $\frac{2}{p} + \frac{2}{q} + \frac{2}{r}$

(ii) $(p - 4)(q - 4)(r - 4)$

(iii) $p^3 + q^3 + r^3$

(8)

(Total 8 marks)

Edexcel A2 June 2019 Paper 2 Q2

Solutions

1.

Scheme	Marks	AO
Circles correct answer	B1	1.1b
	(1 mark)	

Typical solution

$$p^2 + 12$$

2.

Scheme	Marks	AO
Circles the correct answer	B1	1.1b
	(1 mark)	

Typical solution

$$-\frac{3}{5}$$

3.

Scheme	Marks	AO
DR $\alpha + \beta = k, \alpha\beta = 2k$	B1	1.1a
$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$	M1	2.3
$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1	3.1a
$= \frac{k^2 - 4k}{2k} = \frac{1}{2}k - 2$	A1	1.1
	[4]	

Notes

M1: (1st) for combining fractions correctly

M1: (2nd) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ (soi)

A1: must be simplified (accept $\frac{k-4}{2}$)

Alternative solution

Scheme	Marks
$x = \frac{k \pm \sqrt{k^2 - 8k}}{2}$	B1
$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{k + \sqrt{k^2 - 8k}}{k - \sqrt{k^2 - 8k}} + \frac{k - \sqrt{k^2 - 8k}}{k + \sqrt{k^2 - 8k}}$ $= \frac{(k + \sqrt{k^2 - 8k})^2 + (k - \sqrt{k^2 - 8k})^2}{(k - \sqrt{k^2 - 8k})(k + \sqrt{k^2 - 8k})}$	M1
$= \frac{2(k^2 + k^2 - 8k)}{k^2 - k^2 + 8k}$	A1
$= \frac{2(2k^2 - 8k)}{8k} = \frac{1}{2}k - 2$	A1

B1: by formula or completing the square

M1: combining fractions

A1: (1st) expanding correctly

A1: (2nd) must be simplified (accept $\frac{k-4}{2}$)

4.

Scheme	Marks	AO
$u = x + 2$	B1	1.1
$(u - 2)^3 = u^3 - 3 \times 2u^2 + 3 \times 2^2u - 2^3$	M1	1.1
$(u - 2)^4 = u^4 - 4 \times 2u^3 + 6 \times 2^2u^2 - 4 \times 2^3u + 2^4$	M1	1.1
$4(u - 2)^4 - 2(u - 2)^3 - 3(u - 2) + 2 (= 0)$	M1	1.1
$\therefore 4(u^4 - 8u^3 + 24u^2 - 32u + 16) - 2(u^3 - 6u^2 + 12u - 8) - 3(u - 2) + 2 = 0$ $\therefore 4u^4 - 34u^3 + 108u^2 - 155u + 88 = 0$	A1	1.1
	[5]	

Notes

B1: Correct substitution stated or used

M1: Attempt to expand their $(u - 2)^3$ following a linear substitution. 4 terms using $\binom{n}{r} 2^r u^{n-r}$
 $u^3 - 6u^2 + 12u - 8$

Binomial coefficients might not be correct or evaluated. Might be by expanding brackets.

M1: Attempt to expand their $(u - 2)^4$ following a linear substitution. 5 terms using $\binom{n}{r} 2^r u^{n-r}$
 $u^4 - 8u^3 + 24u^2 - 32u + 16$

Binomial coefficients might not be correct or evaluated. Might be by expanding brackets.

M1: Forming (LHS of) equation in u (could be using their expansions)

A1: Final answer can be in terms of x

Final answer needs to be an equation with coefficients simplified.

ISW attempts to cancel down following correct equation

5.

Scheme	Marks	AO
DR $\alpha \cdot \frac{1}{\alpha} \cdot \beta = -3$	M1	3.1a
$\Rightarrow \beta = -3$	A1	1.1
$\alpha + \frac{1}{\alpha} + \beta = -2$	M1	1.1
$\Rightarrow \alpha^2 - \alpha + 1 = 0$	M1	1.1
$\Rightarrow \alpha = \frac{1}{2} + \frac{\sqrt{3}}{2}i \text{ or } \frac{1}{2} - \frac{\sqrt{3}}{2}i$	A1	1.1
[as $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1,$ $\Rightarrow \alpha = \frac{1}{2} + \frac{\sqrt{3}}{2}i \Rightarrow \frac{1}{\alpha} = \frac{1}{2} - \frac{\sqrt{3}}{2}i]$		
so roots are $-3, \frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$	A1	3.2a
	[6]	

Notes

M1: (1st) product of roots = -3 (condone 3 for M1)

M1: (2nd) sum of roots = -2 (condone 2 for M1)

M1: (3rd) getting quadratic in α or $k = -2$ found $\Rightarrow z^2 - z + 1$ by factorising

A1: (2nd) $\Rightarrow z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ or $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

[...] not required for final A1

(corrected from the printed mark scheme: the last line printed only “so roots are $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ ”, leaving out the third root $\beta = -3$ found above)

6.

Mark scheme (i)

Scheme	Marks	AO
$p + q + r = 2, \quad pq + pr + qr = 4, \quad pqr = 5$	B1	3.1a
$\frac{2}{p} + \frac{2}{q} + \frac{2}{r} = \frac{2(pq + pr + qr)}{pqr}$	M1	1.1b
$= \frac{8}{5}$	A1ft	1.1b
	(3)	

Notes

B1: Identifies the correct values for all 3 expressions (can score anywhere). Allow notation such as $\sum p$, $\sum pq$ for the sum and pair sum.

M1: Uses a correct identity for the sum

A1ft: Correct value (follow through their 2, 4 and 5)

Alternative for part (i)

Scheme	Marks	AO
$x = \frac{2}{y} \Rightarrow \frac{8}{y^3} - \frac{8}{y^2} + \frac{8}{y} - 5 = 0 \Rightarrow 5y^3 - 8y^2 + 8y - 8 = 0$	B1	3.1a
$\frac{2}{p} + \frac{2}{q} + \frac{2}{r} = -\frac{8}{5}$	M1	1.1b
$= \frac{8}{5}$	A1ft	1.1b
	(3)	

B1: Obtains the correct cubic in “y”

M1: Uses a correct method

A1ft: Correct value (follow through their 2, 4 and 5)

Mark scheme (ii)

Scheme	Marks	AO
$(p - 4)(q - 4)(r - 4) = (pq - 4p - 4q + 16)(r - 4)$	M1	1.1b
$= pqr - 4pq - 4pr - 4qr + 16p + 16q + 16r - 64$	A1	1.1b
$(= pqr - 4(pq + pr + qr) + 16(p + q + r) - 64)$		
$= 5 - 4(4) + 16(2) - 64 = -43$	A1	1.1b
	(3)	

Notes

M1: Attempt to expand – must have an expression that involves the sum, pair sum and product

A1: Correct expansion

A1: Correct value

Alternative for part (ii)

Scheme	Marks	AO
$(x + 4)^3 - 2(x + 4)^2 + 4(x + 4) - 5 = 0$	M1	1.1b
$= \dots 64 + \dots - 32 + \dots 16 + \dots - 5 = 43$	A1	1.1b
$\therefore (p - 4)(q - 4)(r - 4) = -43$	A1	1.1b
	(3)	

M1: Substitutes $x + 4$ for x in the given cubic

A1: Calculates the correct constant term

A1: Correct value

Mark scheme (iii)

Scheme	Marks	AO
E.g. $p^3 + q^3 + r^3 =$ $= (p + q + r)^3 - 3(p + q + r)(pq + pr + qr) + 3pqr$ or $= (p + q + r)((p + q + r)^2 - 2(pq + pr + qr) - pq - pr - qr) + 3pqr$	M1	3.1a

or $= 2((p + q + r)^2 - 2(pq + pr + qr)) - 4(p + q + r) + 3pqr$ $\Rightarrow p^3 + q^3 + r^3 = \dots$		
$= 2^3 - 3(2)(4) + 3(5) = -1$ $= 2(2^2 - 3(4)) + 3(5) = -1$ $= 2(2^2 - 2(4)) - 4(2) + 3(5) = -1$	A1	1.1b
	(2)	
	(8 marks)	

Notes

M1: Establishes a correct identity that is in terms of the sum, pair sum and product and substitutes to reach a numerical expression for $p^3 + q^3 + r^3$

A1: Correct value

Alternative for part (iii)

Scheme	Marks	AO
$p^3 - 2p^2 + 4p - 5 = 0$, $q^3 - 2q^2 + 4q - 5 = 0$, $r^3 - 2r^2 + 4r - 5 = 0$ $p^3 + q^3 + r^3 - 2(p^2 + q^2 + r^2) + 4(p + q + r) - 15 = 0$ $p^3 + q^3 + r^3 = 2((p + q + r)^2 - 2(pq + pr + qr)) - 4(p + q + r) + 15$ $\Rightarrow p^3 + q^3 + r^3 = \dots$	M1	3.1a
$= 2(2^2 - 2(4)) - 4(2) + 15 = -1$	A1	1.1b
	(2)	