

QQQ – Core Pure 1 – Chapter 4 – Roots of Polynomials (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The quadratic equation $x^2 + px + q = 0$ has roots α and β

Which of the following is equal to $\alpha\beta$?

Circle your answer.

[1 mark]

- p
- $-p$
- q
- $-q$

(Total 1 marks)

AQA AS June 2022 Paper 1 Q2

2. The quartic equation

$$3x^4 + x^2 - 5x - 11 = 0$$

has roots α, β, γ and δ

(a) Write down the value of $\alpha\beta\gamma\delta$

[1 mark]

(b) Write down the value of $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

[1 mark]

(Total 2 marks)

AQA AS June 2025 Paper 1 Q5

3. The roots of the equation $2x^2 + 3x + 5 = 0$ are denoted by α and β .

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

[2]

(b) Using the answers to part (a) determine the value of each of the following.

- $\alpha^2 + \beta^2$
- $\frac{1}{\alpha} + \frac{1}{\beta}$

[4]

(Total 6 marks)

OCR A AS June 2025 Paper 1 Q3

4. The equation $2x^3 - 2x^2 + 8x - 15 = 0$ has roots α, β and γ .

Determine the value of $\alpha^2 + \beta^2 + \gamma^2$.

[4]

(Total 4 marks)

OCR MEI A2 June 2024 Paper 1 Q3

5. The cubic equation

$$z^3 - 3z^2 + z + 5 = 0$$

has roots α, β and γ .

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

(Total 5 marks)

Edexcel AS June 2018 Paper 1 Q2

6. The three roots of the equation

$$4x^3 - 12x^2 - 13x + k = 0$$

where k is a constant, form an arithmetic sequence.

Find the roots of the equation.

[6 marks]

(Total 6 marks)

AQA A2 June 2020 Paper 1 Q8

Solutions

1.

Scheme	Marks	AO
Circles the correct answer.	B1	1.2
	(1 mark)	

Typical solution

$$q$$

2.

Mark scheme (a)

Scheme	Marks	AO
Obtains $-\frac{11}{3}$	B1	1.1b
	(1)	

Typical solution

$$\alpha\beta\gamma\delta = -\frac{11}{3}$$

Mark scheme (b)

Scheme	Marks	AO
Obtains $\frac{5}{3}$	B1	1.1b
	(1)	
	(2 marks)	

Typical solution

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{5}{3}$$

3.

Mark scheme (a)

Scheme	Marks	AO
$\alpha + \beta = -\frac{3}{2}$	B1	1.1
$\alpha\beta = \frac{5}{2}$	B1	1.1
	[2]	

Mark scheme (b)

Scheme	Marks	AO
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1	3.1a
$= \left(-\frac{3}{2}\right)^2 - 2\left(\frac{5}{2}\right) = \frac{9}{4} - \frac{20}{4} = -\frac{11}{4}$ oe	A1	1.1
$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$	M1	1.1
$= \frac{\left(-\frac{3}{2}\right)}{\left(\frac{5}{2}\right)} = -\frac{3}{5}$ oe	A1	1.1
	[4]	

Notes

M1: Expressing $\alpha^2 + \beta^2$ in terms of the symmetric functions

“Using answers to part a”, so some evidence of how these are used is needed

A1: -2.75

M1: Expressing $1/\alpha + 1/\beta$ in terms of the symmetric functions

A1: Exact equivalent only (-0.6)

4.

Scheme	Marks	AO
$\alpha + \beta + \gamma = 1$	B1	3.1a
$\alpha\beta + \beta\gamma + \gamma\alpha = 4$	B1	1.1
$[\alpha^2 + \beta^2 + \gamma^2 =](\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$	M1	3.1a

$= -7$	A1	1.1
	[4]	

Notes

B1: May be embedded

B1: May be embedded

M1: oe Accept numerical values in terms of their 1 and 4, i.e. $1^2 - 2 \times 4$. No slips allowed.

Alternative method

Scheme	Marks
Let $z = x^2$ $2(\sqrt{z})^3 - 2(\sqrt{z})^2 + 8\sqrt{z} - 15 = 0$	M1
$(\sqrt{z})^2(2z + 8)^2 = (15 + 2z)^2$	M1
$4z^3 + 28z^2 + 4z - 225 = 0$	A1
$\alpha^2 + \beta^2 + \gamma^2 = -7$	A1
	[4]

M1: Correct substitution chosen and attempted

M1: Rearranging and squaring both sides to remove square root

A1: Correct equation

5.

Scheme	Marks	AO
$w = 2z + 1 \Rightarrow z = \frac{w-1}{2}$	B1	3.1a
$\left(\frac{w-1}{2}\right)^3 - 3\left(\frac{w-1}{2}\right)^2 + \left(\frac{w-1}{2}\right) + 5 = 0$	M1	3.1a
$\frac{1}{8}(w^3 - 3w^2 + 3w - 1) - \frac{3}{4}(w^2 - 2w + 1) + \frac{w-1}{2} + 5 = 0$		
$w^3 - 9w^2 + 19w + 29 = 0$	M1 A1 A1	1.1b 1.1b 1.1b

	(5)	
	(5 marks)	

Notes

B1: Selects the method of making a connection between z and w by writing $z = \frac{w-1}{2}$

M1: Applies the process of substituting their $z = \frac{w-1}{2}$ into $z^3 - 3z^2 + z + 5 = 0$

(Allow $z = 2w + 1$)

M1: Manipulates their equation into the form $w^3 + pw^2 + qw + r (= 0)$ having substituted their z in terms of w . Note that the “= 0” can be missing for this mark.

A1: At least two of p, q, r correct. Note that the “= 0” can be missing for this mark.

A1: Fully correct equation including “= 0”

The first 4 marks are available if another letter is used instead of w but the final answer must be in terms of w .

Alternative (ALT 1)

Scheme	Marks	AO
$\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \alpha\gamma = 1, \alpha\beta\gamma = -5$	B1	3.1a
New sum $= 2(\alpha + \beta + \gamma) + 3 = 9$ New pair sum $= 4(\alpha\beta + \beta\gamma + \gamma\alpha) + 4(\alpha + \beta + \gamma) + 3 = 19$ New product $= 8\alpha\beta\gamma + 4(\alpha\beta + \beta\gamma + \gamma\alpha) + 2(\alpha + \beta + \gamma) + 1 = -29$	M1	3.1a
$w^3 - 9w^2 + 19w + 29 = 0$	M1 A1 A1	1.1b 1.1b 1.1b
	(5)	

B1: Selects the method of giving three correct equations containing α, β and γ

M1: Applies the process of finding the new sum, new pair sum, new product

M1: Applies $w^3 - (\text{new sum})w^2 + (\text{new pair sum})w - (\text{new product}) (= 0)$

or identifies p as $-(\text{new sum})$ q as (new pair sum) and r as $-(\text{new product})$

A1: At least two of p, q, r correct.

A1: Fully correct equation including “= 0”

The first 4 marks are available if another letter is used instead of w but the final answer must be in terms of w .

6.

Scheme	Marks	AO
Defines three roots in arithmetic progression. PI by a correct use of arithmetic series sum in terms of α and β	M1	3.1a
Uses one of Vieta's laws: $\alpha + \beta + \gamma = 3$ $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{-13}{4}$ $\alpha\beta\gamma = \frac{-k}{4}$	M1	3.1a
Obtains 1 as a root of the equation (this might be $\alpha + D = 1$ without clear realisation that 1 is therefore a root)	A1	1.1b
Substitutes their root of 1 in the cubic equation to find k or uses another of Vieta's laws with their root of 1 substituted.	M1	3.1a
Obtains the value of k or solves their equation to find D or the other roots. Allow one slip.	A1F	1.1b
Correctly obtains all three roots: $-1.5, 1, 3.5$	A1	1.1b
	(6 marks)	

Typical solution

Let the roots be $\alpha - D, \alpha$ and $\alpha + D$

$$\Sigma\alpha: \quad \alpha - D + \alpha + \alpha + D = -\left(\frac{-12}{4}\right)$$

$$3\alpha = 3$$

$$\alpha = 1$$

$\Sigma\alpha\beta:$

$$(1 - D)(1) + (1)(1 + D) + (1 + D)(1 - D) = \frac{-13}{4}$$

$$1 - D + 1 + D + 1 - D^2 = \frac{-13}{4}$$

$$3 + \frac{13}{4} = D^2 \Rightarrow D = \pm \frac{5}{2}$$

$$x = -1.5, 1, 3.5$$