

QQQ – Core Pure 1 – Chapter 3 – Series (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Using the formulae for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$, show that $\sum_{r=1}^{10} r(3r - 2) = 1045$. **[3]**

(Total 3 marks)

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2. Find $\sum_{r=1}^n (2r^2 - 1)$, expressing your answer in fully factorised form. **[4]**

(Total 4 marks)

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3. In this question you must show detailed reasoning.

Determine the sum of all cube numbers from 216 to 512 000 inclusive. **[4]**

(Total 4 marks)

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4. You are given that $\sum_{r=1}^n (ar + b) = n^2$ for all n , where a and b are constants.

By finding $\sum_{r=1}^n (ar + b)$ in terms of a , b and n , determine the values of a and b . **[6]**

(Total 6 marks)

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5.

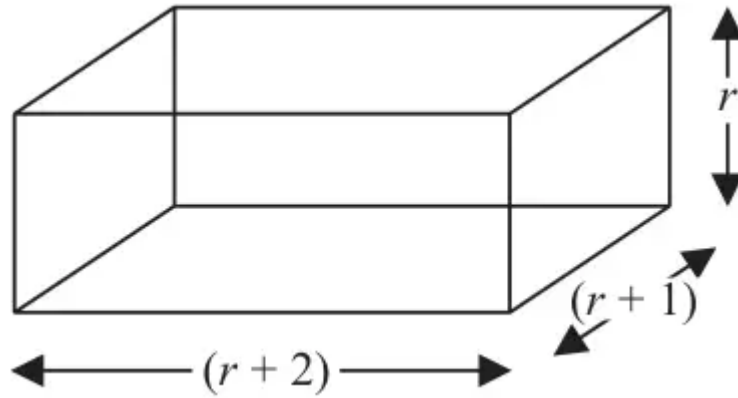


Figure 2

A block has length $(r + 2)$ cm, width $(r + 1)$ cm and height r cm, as shown in Figure 2.

In a set of n such blocks, the first block has a height of 1 cm, the second block has a height of 2 cm, the third block has a height of 3 cm and so on.

- (a) Use the standard results for $\sum_{r=1}^n r^3$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r$ to show that the **total** volume, V , of all n blocks in the set is given by

$$V = \frac{n}{4}(n + 1)(n + 2)(n + 3) \quad n \geq 1$$

(5)

Given that the total volume of all n blocks is

$$(n^4 + 6n^3 - 11\,710) \text{ cm}^3$$

- (b) determine how many blocks make up the set.

(2)

(Total 7 marks)

Solutions

1.

Scheme	Marks	AO
$\sum_{r=1}^{10} r(3r - 2) = \sum_{r=1}^{10} (3r^2 - 2r) = 3 \sum_{r=1}^{10} r^2 - 2 \sum_{r=1}^{10} r$	M1	1.1
$= 3 \left(\frac{1}{6} 10 \cdot 11 \cdot 21 \right) - 2 \left(\frac{1}{2} 10 \cdot 11 \right)$	M1	2.1
$(= 55(21 - 2) = 55 \times 19)$ $= 1045$	A1	1.1
	[3]	

Notes

M1: Separate **soi**

M1: Use both formulae with $n = 10$

A1: **AG. oe** 1155 - 110

Alternative method (leaving substitution to the end)

Scheme	Marks
$\sum_{r=1}^{10} r(3r - 2) = \sum_{r=1}^{10} (3r^2 - 2r) = 3 \sum_{r=1}^{10} r^2 - 2 \sum_{r=1}^{10} r$	M1
$= \frac{3}{6} n(n+1)(2n+1) - \frac{2}{2} n(n+1)$ $= \frac{1}{2} n(n+1)(2n+1-2) = \frac{1}{2} n(n+1)(2n-1)$ $n = 10 \Rightarrow \frac{1}{2} 10 \cdot 11 \cdot 19$	M1
$= 1045$	A1
	[3]

2.

Scheme	Marks	AO
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$\sum_{r=1}^n (2r^2 - 1) = \frac{1}{3}n(n+1)(2n+1) - n$	B1 B1	2.5 1.1b
$= \frac{1}{3}n(2n^2 + 3n - 2)$	B1	1.1b
$= \frac{1}{3}n(2n-1)(n+2)$	B1cao	1.1b
	[4]	

Notes

B1: $\frac{1}{3}n(n+1)(2n+1) \dots$

B1: $\dots - n$

B1: factoring out n correctly

B1cao: allow $\frac{2}{6}n(2n-1)(n+2)$, etc

3.

Scheme	Marks	AO
DR $(\sqrt[3]{512000} =) 80$	B1	3.1a
$\sum_{r=1}^{80} r^3 = \frac{1}{4} \times 80^2 \times (80+1)^2$	M1	1.1
$= 10497600$ OR $-\frac{1}{4} \times 5^2 \times 6^2$ OR -225	A1	1.1
Required sum = 10497375	A1	2.2a
	[4]	

Notes

B1: May be seen embedded in a sum

M1: Using the correct formula for sum of cubes with an upper limit (between 26-720 but not 216) substituted

A1: Any one of these seen correct

May see $1 + 8 + 27 + 64 + 125 = 225$

A1: Correct answer with no working is 0/4

Alternative method

Scheme	Marks
DR $(\sqrt[3]{512000} = 80 \text{ and } 80 - 5 =) 75$	B1
Sum is $\left(\sum_{r=6}^{80} r^3\right) = \sum_{R=1}^{75} (R+5)^3 = \sum_{R=1}^{75} R^3 + 15R^2 + 75R + 125$	M1
$= \frac{1}{4} \times 75^2 \times (75+1)^2$ $+ 15 \times \frac{75(75+1)(2 \times 75+1)}{6} + 75 \times \frac{75(75+1)}{2}$ $+ 125 \times 75$	A1
$= 8122500 + 2151750 + 213750 + 9375$ $= 10497375$	A1

B1: May be seen embedded in a sum (e.g. 75 as upper limit)

M1: Expressing the required sum as a single sum and multiplying out so that standard formulae can be applied (allow one slip in coefficients). Ignore limits for this mark.

A1: 3 out of 4 terms correct (must include 125×75)

A1: Correct answer with no working is 0/4

4.

Scheme	Marks	AO
$\sum_{r=1}^n (ar + b) = a \sum_{r=1}^n r + b \sum_{r=1}^n 1$	M1	1.1a
$\frac{1}{2}an(n+1) + bn$	A1 A1	1.2 1.1
$\frac{1}{2}an(n+1) + bn = n^2$ $\frac{1}{2}a = 1, \frac{1}{2}a + b = 0$	M1	3.1a
So $a = 2$	A1	1.1
$b = -1$	A1	1.1
	[6]	

Notes

M1: (1st) splitting sum

A1 A1: $\frac{1}{2}an(n+1) \dots$

$\dots + bn$

M1: (2nd) equating coefficients (or substituting two values for n)

Alternative solution

Scheme	Marks
$\sum_{r=1}^n (ar + b)$ is an AP with 1 st term $a + b$, common diff a	M1
so $\sum_{r=1}^n (ar + b) = \frac{n}{2}[2(a + b) + (n - 1)a]$	M1A1
$= \frac{n}{2}[2a + 2b + na - a] = \frac{1}{2}na + nb + \frac{1}{2}n^2a$ $\frac{1}{2}a = 1, \frac{1}{2}a + b = 0$	M1
So $a = 2$	A1
$b = -1$	A1

M1A1: use of sum formula (need not be simplified)

M1: (3rd) equating coeffs or substituting two values for n

Alternative solution

Scheme	Marks
substitute $n = 1: a + b = 1$ substitute $n = 2: 3a + 2b = 4$ solving simultaneously: $2a + 2b = 2$	M1
so $a = 2$	A1
$b = -1$	A1

As the question requires finding $\sum_{r=1}^n (ar + b)$ in terms of a, b and n , allow a maximum of 3 marks for this method

5.

Mark scheme (a)

Scheme	Marks	AO
Volume = $r \times (r + 1) \times (r + 2)$	B1	1.1b

A complete method for finding the total volume of n blocks and expressing it in sigma notation. This can be implied by later work. $\sum_{r=1}^n (r^3 + 3r^2 + 2r)$	M1	3.1b
$V = \frac{1}{4}n^2(n+1)^2 + 3 \times \frac{1}{6}n(n+1)(2n+1) + 2 \times \frac{n}{2}(n+1)$	M1	2.1
$V = \frac{1}{4}n(n+1)[n(n+1) + 2(2n+1) + 4]$	dM1	1.1b
$V = \frac{1}{4}n(n+1)[n^2 + 5n + 6]$ $\Rightarrow V = \frac{1}{4}n(n+1)(n+2)(n+3)^*$	A1*	1.1b
	(5)	

Notes

B1: Correct volume of a block

M1: Expressing the total volume of all n blocks as a series in terms of r , r^2 and r^3

M1: Substitutes at least one of the standard formulae into their volume.

dM1: Attempts to factorise $\frac{1}{4}n(n+1)$ having used at least one standard formula correctly. Each term must contain a factor of $n(n+1)$

A1*: Obtains the printed result with no errors seen, no bracketing errors and following from

$$V = \frac{1}{4}n(n+1)[n^2 + 5n + 6] \text{ o.e.}$$

Note: Going from $\frac{1}{4}n(n^3 + 6n^2 + 11n + 6)$ to $\frac{1}{4}n(n+1)(n+2)(n+3)$ with no reasoning shown scores **dM0 A0**

Mark scheme (b)

Scheme	Marks	A0
Sets $\frac{1}{4}n(n+1)(n+2)(n+3) = n^4 + 6n^3 - 11710$ $\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{11}{4}n^2 + \frac{3}{2}n = n^4 + 6n^3 - 11710$ simplifies $(3n^4 + 18n^3 - 11n^2 - 6n - 46840 = 0)$ and solves $n = \dots$	M1	1.1b

There are 10 blocks or $n = 10$	A1	3.2a
	(2)	
	(7 marks)	

Notes

M1: Sets the printed answer $= n^4 + 6n^3 - 11710$, simplifies, collects terms and uses their calculator to solve a quartic equation to find a value for n .

A1: Selects $n = 10$ or states that there are **10 blocks** from a **correct equation**