

QQQ – Core Pure 1 – Chapter 3 – Series (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Find

$$\sum_{r=1}^{20} (r^2 - 2r)$$

Circle your answer.

[1 mark]

- 2450
- 2660
- 5320
- 43680

(Total 1 marks)

AQA A2 June 2021 Paper 1 Q1

2. The positive integer k is such that

$$\sum_{r=1}^k (3r - k) = 90$$

Find the value of k .

[3 marks]

(Total 3 marks)

AQA A2 June 2019 Paper 2 Q4

3. Using standard summation formulae, find $\sum_{r=1}^n (r^2 - 3r)$, giving your answer in fully factorised form.

[3]

(Total 3 marks)

OCR MEI AS October 2021 Paper 1 Q1

4. Show that

$$\sum_{r=11}^{n+1} r^3 = \frac{1}{4} (n^2 + an + b) (n^2 + an + c)$$

where a , b and c are integers to be found.

[3 marks]

(Total 3 marks)

AQA A2 June 2023 Paper 2 Q7

5. In this question you must show detailed reasoning.

The series S is defined as being the sum of the squares of all positive **odd** integers from 1^2 to 779^2 .

Determine the value of S .

[5]

(Total 5 marks)

OCR A A2 June 2024 Paper 2 Q4

6. An art display consists of an arrangement of n marbles.

When arranged in ascending order of mass, the mass of the first marble is 10 grams.

The mass of each subsequent marble is 3 grams more than the mass of the previous one, so that the r th marble has mass $(7 + 3r)$ grams.

(a) Show that the mean mass, in grams, of the marbles in the display is given by

$$\frac{1}{2}(3n + 17)$$

(3)

Given that there are 85 marbles in the display,

(b) use the standard summation formulae to find the standard deviation of the mass of the marbles in the display, giving your answer, in grams, to one decimal place.

(6)

(Total 9 marks)

Edexcel AS June 2019 Paper 1 Q6

Solutions

1.

Scheme	Marks	AO
Circles correct answer	B1	1.1b
	(1 mark)	

Typical solution

2450

2.

Scheme	Marks	AO
Removes $\sum$ sign and creates an expression for the sum in terms of k . Allow max. 1 error	M1	1.1a
Forms a correct quadratic equation	A1	1.1b
Obtains the correct answer. NMS 3/3 Treat trial and improvement as NMS	A1	1.1b
	(3 marks)	

Typical solution

$$\sum_{r=1}^k (3r - k) = \frac{3}{2}k(k+1) - k^2$$

$$\frac{1}{2}k^2 + \frac{3}{2}k = 90$$

$$k = 12$$

3.

Scheme	Marks	AO
--------	-------	----

$\sum_{r=1}^n (r^2 - 3r) = \sum_{r=1}^n r^2 - 3 \sum_{r=1}^n r$	M1	1.1a
$= \frac{1}{6}n(n+1)(2n+1) - \frac{3}{2}n(n+1)$	A1	1.1
$= \frac{1}{3}n(n+1)(n-4)$	A1	1.1
	[3]	

4.

Scheme	Marks	AO
Expresses the sum as the difference of two series.	M1	3.1a
Obtains a correct (unsimplified) expression in terms of n for the sum.	A1	1.1b
Obtains the required result.	R1	2.1
	(3 marks)	

Typical solution

$$\begin{aligned}
 \sum_{r=11}^{n+1} r^3 &= \sum_{r=1}^{n+1} r^3 - \sum_{r=1}^{10} r^3 \\
 &= \frac{1}{4}(n+1)^2(n+2)^2 - \frac{1}{4}(10)^2(11)^2 \\
 &= \frac{1}{4} \left\{ ((n+1)(n+2) + 110)((n+1)(n+2) - 110) \right\} \\
 &= \frac{1}{4} \left\{ (n^2 + 3n + 112)(n^2 + 3n - 108) \right\}
 \end{aligned}$$

5.

Scheme	Marks	AO
$S = \sum_{r=1}^{390} (2r-1)^2$ or $S = \sum_{r=0}^{389} (2r+1)^2$	M1	3.1a
$= \sum_{r=1}^{390} (4r^2 - 4r + 1) = 4 \sum_{r=1}^{390} r^2 - 4 \sum_{r=1}^{390} r + \sum_{r=1}^{390} 1$ or $= \sum_{r=0}^{389} (4r^2 + 4r + 1) = 4 \sum_{r=0}^{389} r^2 + 4 \sum_{r=0}^{389} r + \sum_{r=0}^{389} 1$	M1	2.2a

$\sum_{r=1}^{390} r^2 = \frac{1}{6} \times 390(390 + 1)(2 \times 390 + 1)$ or 19 849 115 or 79 396 460	B1FT	1.1
$\sum_{r=1}^{390} r = \frac{390(390 + 1)}{2} \text{ or } 76\,245 \text{ or } 304\,980$ and $\sum_{r=1}^{390} 1 = 390$	B1FT	1.1
So $S = 4 \times 19\,849\,115 - 4 \times 76\,245 + 390$ $= 79\,091\,870$	A1	1.1
	[5]	

Notes

M1: DR. Correctly setting up S as a sum of odd squares (condone incorrect limit combination: allow 0 or 1 for lower and 389 or 390 or 399 or 400 for upper). Treat 399 or 400 as MR (799 for 779).

M1: Expanding and separating (with their limits). Term “1” must be dealt with correctly although condone poor notation if their 390 appropriately seen later). **MOM1** is possible

B1FT: FT only 389 or 390. Correctly using the formula for the sum of squares up to and including 390 (or 389)

or 19 697 015 from 0 or 1 up to 389

B1FT: FT only 389 or 390. Correctly using the formula for the sum of integers up to and including 390 (or 389) and “sigma 1 = n ” leading to their n .

or 75 855 from 0 or 1 up to 389

NB **B1B1** can only be attained with consistent upper limits

Alternative method

Scheme	Marks
$S = 1^2 + 3^2 + 5^2 + \dots + 779^2$ $= \sum_{r=1}^{779} r^2 - (2^2 + 4^2 + 6^2 + \dots + 778^2)$	M1
$\sum_{r=1}^{779} r^2 = \frac{1}{6} \times 779(779 + 1)(2 \times 779 + 1)$ (= 157 879 930)	B1

$S = \sum_{r=1}^{779} r^2 - (2^2 + 4^2 + 6^2 + \dots + 778^2)$ $= \sum_{r=1}^{779} r^2 - 4(1^2 + 2^2 + 3^2 + \dots + 389^2)$	M1
$\sum_{r=1}^{389} r^2 = \frac{1}{6} \times 389(389 + 1)(2 \times 389 + 1)$ $(= 19\,697\,015)$	B1
So $S = 157\,879\,930 - 4 \times 19\,697\,015$ $= 79\,091\,870$	A1
	[5]

M1: Correctly setting up S and writing it as the difference between two sums of squares.

or
$$\sum_{r=1}^{780} r^2 - (2^2 + 4^2 + 6^2 + \dots + 780^2)$$

B1: Correctly using the formula for the sum of squares up to and including 779 (or 780)
or 158 488 330 up to 780

M1: Bringing out a factor of 2^2 (or 4) to convert the sum of even squares to a sum of squares. Could be separate.

B1: Correctly using the formula for the sum of squares up to and including 389 (or 390)
or 19 849 115 up to 390

NB **B1B1** can only be attained for 779 and 389 or 780 and 390

A1: $158\,488\,330 - 4 \times 19\,849\,115$

6.

Mark scheme (a)

Scheme	Marks	AO
$(\text{mean} = \bar{x} =) \frac{1}{n} \sum_{r=1}^n (7 + 3r)$	M1	1.1a
$\sum_{r=1}^n (7 + 3r) = \left(7 \sum_{r=1}^n 1 + 3 \sum_{r=1}^n r \right) = 7n + 3 \frac{n}{2}(n + 1)$	M1	1.1b
$\bar{x} = 7 + \frac{3}{2}(n + 1) = \frac{14 + 3n + 3}{2} = \frac{1}{2}(3n + 17) *$	A1*	2.1
	(3)	

Notes

M1: Selects the correct procedure for finding the mean ($\bar{x}$), attempting sum and dividing by n .

M1: Splits the sum and applies the formulae for $\sum r$ (accept $7 + 3\frac{n}{2}(n + 1)$ here)

Or uses arithmetic series formula $\frac{1}{2}n(a + l)$ with $a = 10$ and l an attempt at $7 + 3 \times n$, or $\frac{n}{2}(2a + (n - 1)d)$ with $a = 10$ and $d = 3$.

A1*: Correct work proceeding to the answer with an intermediate step shown.

Special case: Award M0M1A0 for candidates who use $\frac{1}{2}(a + l)$ or equivalent without justification of the division by n .

Mark scheme (b)

Scheme	Marks	AO
Correct overall strategy to find the variance or standard deviation. This must include: - An attempt to find the mean - An attempt at $\sum(7 + 3r)^2$ as part of their formula (however poor, or if stated and followed by a value or if used with incorrect limits). - An attempt at either variance formula with their mean (allow slips in the formula)	M1	3.1a
(Mean) mean ($= \bar{x}$) = 136	B1	1.1b
(Sum) Way 1: $\sum_{r=1}^n (7 + 3r)^2 = \sum_{r=1}^n (49 + 42r + 9r^2)$ $= \underline{\underline{49n}} + \underline{\underline{42 \times \frac{1}{2}n(n + 1)}} + \underline{\underline{9 \times \frac{1}{6}n(n + 1)(2n + 1)}}$ Way 2:	M1 B1	1.1b 1.1b

$\sum_{r=1}^n (x_i - \bar{x})^2 = \sum_{r=1}^n (7 + 3r - "136")^2 = a \sum_{r=1}^n r^2 + b \sum_{r=1}^n r + c \sum_{r=1}^n 1$ $= \underline{9 \times \frac{1}{6}n(n+1)(2n+1)} - \underline{"774" \times \frac{1}{2}n(n+1)} + \underline{"16641"n}$		
(Variance/standard deviation) Way 1: $= \frac{"2032690"}{85} - 136^2 = \dots \quad \text{or} \quad \frac{"2032690"}{84} - \frac{85}{84} \times 136^2 = \dots$ Way 2: $= \frac{"460530"}{85} = \dots \quad \text{or} \quad \frac{"460530"}{84} = \dots$ (using sample standard deviation).	M1	1.1b
So s.d. = $\sqrt{5418} = 73.6$ (g) Accept 74.0 (g) if sample s.d. used	A1	1.1b
	(6)	
	(9 marks)	

Notes

(In the scheme, the single-underlined terms are for the M1 and the double-underlined term is for the B1.)

M1: Correct overall strategy to get as far as the variance of marbles in the collection. The attempt at variance should be recognisable (though allow e.g. sign slips in the formula for this mark) and an attempt, however poor, at $\sum (7 + 3r)^2$ must have been made

B1: Correct value for the mean for 85 marbles (accept as a single fraction, $\frac{272}{2}$). If a student works algebraically until the last step, a correct final answer will imply this mark.

M1: Expands brackets and applies summation formulae for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$ to their expression, either in terms of n or with $n = 85$ **but must have correct limits**. Allow for obtaining an expression of the correct form for Way 2 if the mean is kept in terms of " n ".

This mark is for correct application of these two summation formula on an attempt at $\sum_{r=1}^n (7 + 3r)^2$ so accept even if this is not part of an attempt at the variance.

B1: Correct use of $\sum_{r=1}^n 1 = n$ in their expression (must be correct limits).

M1: Correctly applies variance or standard deviation formula with $n = 85$, their attempt at $\sum x^2$ (which need not be using $7 + 3r$ or correct limits) and their mean. Accept use of the sample variance/standard deviation (dividing by $n - 1$)

For reference the variance formula is

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 = \left(\frac{1}{n} \sum_{i=1}^n x_i^2 \right) - \bar{x}^2$$

where $x_r = 7 + 3r$ here, or accept for sample variance

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 = \left(\frac{1}{n-1} \sum_{i=1}^n x_i^2 \right) - \frac{n\bar{x}^2}{n-1}$$

A1: Correct standard deviation to 1 decimal place. If sample standard deviation is used, the answer will be 74.0 g to 1 d.p. (74.04...)

Note: Question specifies use of summation formula and so these must be seen for the 2nd M and 2nd B mark. However, if just 2032690 appears from a calculator all other marks are available.