

QQQ – Core Pure 1 – Chapter 1 – Complex Numbers (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The equation $x^2 + 4x + c = 0$ has non-real roots.

Find the range of possible values of c

Circle your answer.

[1 mark]

- $c < 4$
- $c \leq 4$
- $c \geq 4$
- $c > 4$

(Total 1 marks)

AQA A2 June 2025 Paper 1 Q1

2. The complex number $z = (3 + 4i)(5 + ci)$, where c is an integer.

It is given that $\operatorname{Re}(z) = 7$

Find the value of c

[2 marks]

(Total 2 marks)

AQA A2 June 2025 Paper 2 Q5

3.

$$z = 5 - 3i, \quad w = 2 + 2i$$

Express in the form $a + bi$, where a and b are real constants,

(a) z^2 , **(2)**

(b) $\frac{z}{w}$. **(3)**

(Total 5 marks)

Edexcel FP1 January 2011 Q1

4. In this question you must show detailed reasoning.

(a) Solve the equation $2z^2 - 10z + 25 = 0$ giving your answers in the form $a + bi$. [2]

(b) Solve the equation $3\omega - 2 = i(5 + 2\omega)$ giving your answer in the form $a + bi$. [4]

(Total 6 marks)

OCR A AS October 2021 Paper 1 Q6

5. Find a cubic equation with real coefficients, two of whose roots are $2 - i$ and 3 . [5]

(Total 5 marks)

OCR MEI AS June 2018 Paper 1 Q4

6. In this question you must show detailed reasoning.

The equation $x^4 - 7x^3 - 2x^2 + 218x - 1428 = 0$ has a root $3 - 5i$.

Find the other three roots of this equation. [6]

(Total 6 marks)

OCR A AS October 2021 Paper 1 Q3

Solutions

1.

Scheme	Marks	AO
Circles 4th answer.	B1	1.1b
	(1 mark)	

Typical solution

$$c > 4$$

2.

Scheme	Marks	AO
Obtains $15 - 4c$	M1	1.1a
Obtains $c = 2$	A1	1.1b
	(2 marks)	

Typical solution

$$\operatorname{Re}(z) = 15 - 4c$$

$$15 - 4c = 7$$

$$c = 2$$

3.

Mark scheme (a)

Scheme	Marks
$z = 5 - 3i, \quad w = 2 + 2i$ $z^2 = (5 - 3i)(5 - 3i)$	
$= 25 - 15i - 15i + 9i^2$ $= 25 - 15i - 15i - 9$ <i>An attempt to multiply out the brackets to give four terms (or four terms implied). zw is M0</i>	M1

$= 16 - 30i$ $16 - 30i$ Answer only 2/2	A1
	(2)

Mark scheme (b)

Scheme	Marks
$\frac{z}{w} = \frac{(5 - 3i)}{(2 + 2i)}$	
$= \frac{(5 - 3i)}{(2 + 2i)} \times \frac{(2 - 2i)}{(2 - 2i)}$ <i>Multiplies $\frac{z}{w}$ by $\frac{(2 - 2i)}{(2 - 2i)}$</i>	M1
$= \frac{10 - 10i - 6i - 6}{4 + 4}$ <i>Simplifies realising that a real number is needed on the denominator and applies $i^2 = -1$ on their numerator expression and denominator expression.</i>	M1
$= \frac{4 - 16i}{8}$	
$= \frac{1}{2} - 2i$ $\frac{1}{2} - 2i$ or $a = \frac{1}{2}$ and $b = -2$ or equivalent <i>Answer as a single fraction A0</i>	A1
	(3)
	[5]

4.

Mark scheme (a)

Scheme	Marks	A0
$z = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 2 \times 25}}{2 \times 2}$	M1	2.1
$z = \frac{5}{2} \pm \frac{5}{2}i$	A1	1.1
	[2]	

Notes

M1: Correct substitution into formula. If formula quoted allow one slip.

Or completing the square – one slip allowed.

A1: Allow $z = \frac{5 \pm 5i}{2}$ or equivalent fractions

NB: This question required detailed reasoning

Mark scheme (b)

Scheme	Marks	AO
$3\omega - 2 = 5i + 2i\omega \Rightarrow 3\omega - 2i\omega = 2 + 5i$	M1	1.1
$(3 - 2i)\omega = 2 + 5i \Rightarrow \omega = \frac{2 + 5i}{3 - 2i}$	M1	1.1
$\omega = \frac{2 + 5i}{3 - 2i} \times \frac{3 + 2i}{3 + 2i} = \frac{6 + 4i + 15i - 10}{9 + 4}$	M1	2.1
$\omega = -\frac{4}{13} + \frac{19}{13}i$	A1	1.1
	[4]	

Notes

M1: (1st) Expanding and rearranging

Must rearrange to isolate ω terms on one side and other terms on other side

M1: (2nd) Factorising and dividing by two term complex number

NB: This question required detailed reasoning

M1: (3rd) Multiplying top and bottom by conjugate of bottom

Alternative method

Scheme	Marks
$\omega = a + bi \Rightarrow 3a + 3bi - 2 = 5i + 2ai - 2b$	M1
$3a - 2 = -2b$ and $3b = 5 + 2a$	M1
$9a - 6 + 10 + 4a = 0 \Rightarrow a = -\frac{4}{13}$	M1
$\Rightarrow b = \frac{19}{13} \Rightarrow \omega = -\frac{4}{13} + \frac{19}{13}i$	A1

M1: (1st) Assigning real and imaginary parts, to ω expanding and rearranging

M1: (2nd) Comparing real and imaginary parts

M1: (3rd) Using valid algebra to eliminate one unknown and finding the other

5.

Scheme	Marks	AO
third root is $2 + i$	B1	1.2
$2 - i + 2 + i + 3 = 7$	Blft	3.1a
$(2 - i)(2 + i) + (2 + i)3 + 3(2 - i) = 17$	Blft	1.1
$(2 - i)(2 + i)3 = 15$	Blft	1.1
so eqn is $z^3 - 7z^2 + 17z - 15 = 0$	Blcao	1.1
	[5]	

Notes

B1: soi

Blft: (1st) ft their $2 + i$

Blcao: must include ' $= 0$ '
any variable, e.g. x

Alternative

Scheme	Marks
$(z - 2 + i)(z - 2 - i) = z^2 - 4z + 5$	M1 A1
$(z^2 - 4z + 5)(z - 3) = z^3 - 7z^2 + 17z - 15$ so eqn is $z^3 - 7z^2 + 17z - 15 = 0$	M1 A1

M1: (1st) $(z - 2 + i)(z - 2 - i)$

A1: (1st) $= z^2 - 4z + 5$

M1: (2nd) their $(z^2 - 4z + 5) \times (z - 3)$

6.

Scheme	Marks	AO
$3 + 5i$ is a root	B1	1.2
Attempt to expand $(x - (3 + 5i))(x - (3 - 5i))$	M1	1.1

$= x^2 - 6x + 34$ so this must be a factor	A1	2.2a
$x^4 - 7x^3 - 2x^2 + 218x - 1428 =$ $(x^2 - 6x + 34)(x^2 + \dots x - 42)$ or $(x^2 - 6x + 34)(x^2 - x + \dots)$	M1	1.1
$(x^2 - 6x + 34)(x^2 - x - 42)$	A1	1.1
$(x^2 - x - 42) = (x - 7)(x + 6) \Rightarrow$ roots $-6, 7$ (and $3 + 5i$)	A1	1.1
	[6]	

Notes

B1: Need to see statement that $3 + 5i$ is a root.

May happen at end of question

M1: (1st) Attempt to use the conjugate pair to derive a real quadratic

May see $(3 + 5i)(3 - 5i) = 9 + 25 = 34$ and $(3 + 5i) + (3 - 5i) = 6$ instead of expansion

M1: (2nd) Attempt to factorise or divide resulting in x^2 and one other term

NB: This question required detailed reasoning

A1: (3rd) $3 + 5i$ may be mentioned as a root earlier in the solution