

QQQ – Core Pure 1 – Chapter 1 – Complex Numbers (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Calculate the product

$$(3 + i)(2 - i)$$

Circle your answer.

[1 mark]

- $5 - i$
- $5 + i$
- $7 - i$
- $7 + i$

(Total 1 marks)

AQA AS June 2025 Paper 1 Q1

2. In this question you must show detailed reasoning.

(a) Express $\frac{8 + i}{2 - i}$ in the form $a + bi$ where a and b are real. **[2]**

(b) Solve the equation $4x^2 - 8x + 5 = 0$. Give your answer(s) in the form $c + di$ where c and d are real. **[2]**

(Total 4 marks)

OCR A AS June 2024 Paper 1 Q2

3. The complex number z satisfies the equation $z + 2iz^* + 1 - 4i = 0$.

You are given that $z = x + iy$, where x and y are real numbers.

Determine the values of x and y . **[4]**

(Total 4 marks)

OCR MEI A2 June 2025 Paper 1 Q1

4. Given that $2 - 4i$ is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

(a) write down the other root of the equation, (1)

(b) find the value of p and the value of q . (3)

(Total 4 marks)

Edexcel FP1 January 2011 Q4

5. $2 - 3i$ is one root of the equation

$$z^3 + mz + 52 = 0$$

where m is real.

(a) Find the other roots. [3 marks]

(b) Determine the value of m . [2 marks]

(Total 5 marks)

AQA AS June 2018 Paper 1 Q8

6. It is given that $1 - 3i$ is one root of the quartic equation

$$z^4 - 2z^3 + pz^2 + rz + 80 = 0$$

where p and r are real numbers.

(a) Express $z^4 - 2z^3 + pz^2 + rz + 80$ as the product of two quadratic factors with real coefficients. [4 marks]

(b) Find the value of p and the value of r . [2 marks]

(Total 6 marks)

AQA A2 June 2020 Paper 1 Q4

Solutions

1.

Scheme	Marks	AO
Circles the 3rd answer.	B1	1.1b
	(1 mark)	

Typical solution

$$7 - i$$

2.

Mark scheme (a)

Scheme	Marks	AO
DR $\frac{8+i}{2-i} \times \frac{2+i}{2+i} = \frac{16+8i+2i-1}{4+1}$	M1	1.1
$= \frac{15+10i}{5} = 3+2i \text{ cao}$	A1	1.1
	[2]	

Notes

M1: Multiplying top and bottom by the conjugate of the bottom and attempting expansion involving $i^2 = -1$.

Allow appearance of 5 in denominator with no working shown as long as clear what numerator multiplied by.

A1: Answer must be in the requested form.

Must have at least one line of working before answer (DR)

Alternate method

Scheme	Marks
DR $(8+i) = (2-i)(a+bi)$ $8 = 2a+b$ $1 = -a+2b$	M1

$$\Rightarrow a = 3, b = 2$$

$$3 + 2i \text{ cao}$$

A1

M1: Equating to $a + bi$ and rearranging to real and imaginary parts

A1: Answer must be in the requested form.

Must have at least one line of working before answer (DR)

Mark scheme (b)

Scheme	Marks	AO
DR $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 4 \times 5}}{2 \times 4} = \frac{8 \pm \sqrt{-16}}{8} = \frac{8 \pm 4i}{8}$	M1	1.1
$= 1 + \frac{1}{2}i \text{ or } 1 - \frac{1}{2}i$	A1	1.1
	[2]	

Notes

M1: Use of formula and finding the square root of a negative number in terms of i (can be awarded even if error in calculation under square root).

Condone one sign error (eg -8 rather than $-(-8)$ or $(-8)^2 = -64$).

Could also use completing the square. Must get as far as attempting " $x = \dots$ " but might make some slips.

A1: or $1 \pm \frac{1}{2}i$ or $1 \pm 0.5i$ but must be in correct form so eg $\frac{2 \pm i}{2}$ is **A0**

Condone $1 \pm \frac{i}{2}$

3.

Scheme	Marks	AO
$z^* = x - iy$	B1	1.2
$x + iy + 2ix + 2y + 1 - 4i [= 0]$	M1	1.1
$\Rightarrow x + 2y + 1 = 0, y + 2x - 4 = 0$	M1	3.1a
$\Rightarrow x = 3, y = -2$	A1	1.1
	[4]	

Notes

B1: seen or used

M1: $i^2 = -1$ used with their z^* ; soi by correct equating of real and imaginary parts

M1: equating their real and imaginary parts (imaginary component may still be in terms of i), allow a slip including a missing term

A1: www. Accept $[z =] 3 - 2i$. An M1 step must be seen before final answer.

4.

Mark scheme (a)

Scheme	Marks
$z^2 + pz + q = 0, \quad z_1 = 2 - 4i$	
$z_2 = 2 + 4i$ $2 + 4i$	B1
	(1)

Mark scheme (b)

Scheme	Marks
$(z - 2 + 4i)(z - 2 - 4i) = 0$	
$\Rightarrow z^2 - 2z - 4iz - 2z + 4 + 8i + 4iz - 8i + 16 = 0$ <i>An attempt to multiply out brackets of two complex factors and no i^2.</i>	M1
$\Rightarrow z^2 - 4z + 20 = 0$ <i>Any one of $p = -4, q = 20$.</i>	A1
<i>Both $p = -4, q = 20$.</i> $\Rightarrow z^2 - 4z + 20 = 0$ only 3/3	A1
	(3)
	[4]

Notes

(corrected from the printed mark scheme: the second line printed “ $+4 - 8i$ ” for $+4 + 8i$)

Other possible solutions: (a), (b) Way 2

Scheme	Marks
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$z_2 = 2 + 4i$ $2 + 4i$	B1
Product of roots = $(2 - 4i)(2 + 4i)$ $= 4 + 16 = 20$ or $b^2 - 4ac = (8i)^2$ Sum of roots = $(2 - 4i) + (2 + 4i) = 4$ <i>No i². Attempt Sum and Product of roots or Sum and discriminant</i>	M1
$= z^2 - 4z + 20 = 0$ Any one of $p = -4, q = 20$.	A1
Both $p = -4, q = 20$.	A1
	(4)

Other possible solutions: (a), (b) Way 3

Scheme	Marks
$z_2 = 2 + 4i$ $2 + 4i$	B1
$(2 - 4i)^2 + p(2 - 4i) + q = 0$ An attempt to substitute either z_1 or z_2 into $z^2 + pz + q = 0$ and no i ² .	M1
$-12 - 16i + p(2 - 4i) + q = 0$ Imaginary part: $-16 - 4p = 0$ Real part: $-12 + 2p + q = 0$	
$4p = -16 \Rightarrow p = -4$ Any one of $p = -4, q = 20$.	A1
$q = 12 - 2p \Rightarrow q = 12 - 2(-4) = 20$ Both $p = -4, q = 20$.	A1
	(4)

5.

Mark scheme (a)

Scheme	Marks	AO
Correctly identifies the complex conjugate as a root.	B1	1.1b

Forms one of the following equations (or their equivalents) $\alpha + \beta + 2 - 3i = 0$ or $\alpha\beta(2 - 3i) = \pm 52$ or $\alpha\beta + \beta(2 - 3i) + (2 - 3i)\beta = \pm m$ with an equation to find m . Or forms an equation to find m and then solves the cubic for their value of m .	M1	1.1a
Finds correct third root	A1	1.1b

Typical solution

2nd complex root is $2 + 3i$

$$\text{sum of roots} = -\frac{b}{a}$$

$$\therefore \alpha + 2 + 3i + 2 - 3i = 0$$

$$\alpha + 4 = 0$$

$$\alpha = -4$$

Alternative

2nd complex root is $2 + 3i$

$$\text{product of roots} = -\frac{d}{a}$$

$$\therefore \alpha(2 + 3i)(2 - 3i) = -52$$

$$\alpha(4 - 9i^2) = -52$$

$$\alpha = \frac{-52}{13}$$

$$\alpha = -4$$

Mark scheme (b)

Scheme	Marks	AO
Forms a correct equation to find m . May be seen in part (a).	M1	1.1a
Finds correct value of m .	A1	1.1b
	(5 marks)	

Typical solution

$$\therefore (-4)^3 + m(-4) + 52 = 0$$

$$-64 - 4m + 52 = 0$$

$$-12 = 4m$$

$$m = -3$$

Alternative

$$\sum \alpha\beta = \frac{c}{a}$$

$$\therefore (2 - 3i)(2 + 3i) + (-4)(2 - 3i) + (-4)(2 + 3i) = m$$

$$4 + 9 - 8 + 12i - 8 - 12i = m$$

$$m = -3$$

6.**Mark scheme (a)**

Scheme	Marks	AO
Identifies $1 + 3i$ as a second root of the quartic equation.	B1	1.1b
Uses a pair of conjugate roots to find a quadratic factor.	M1	1.1a
Finds one correct quadratic factor.	A1	1.1b
Correctly expresses the quartic as the product of two quadratic factors.	A1	1.1b

Typical solution

$z = 1 + 3i$ is another root.

$z^2 - 2z + 10$ is a factor.

$$(z^2 - 2z + 10)(z^2 + bz + 8) \equiv z^4 - 2z^3 + pz^2 + rz + 80$$

Comparing z^3 -terms gives $b = 0$

$\therefore$ the quartic is

$$(z^2 - 2z + 10)(z^2 + 8)$$

Mark scheme (b)

Scheme	Marks	AO
Substitutes $1 + 3i$ or $1 - 3i$ or one of their roots from their factorisation in (a) into the quartic equation and compares Re and Im parts or compares the coefficients of z^2 and z from the (possibly partial) expansion of their product of quadratics with the given quartic.	M1	1.1a
Finds the correct values of $p = 18$ and $r = -16$	A1	1.1b
	(6 marks)	

Typical solution

$$z^4 - 2z^3 + 18z^2 - 16z + 80 = 0$$

$$p = 18$$

$$r = -16$$