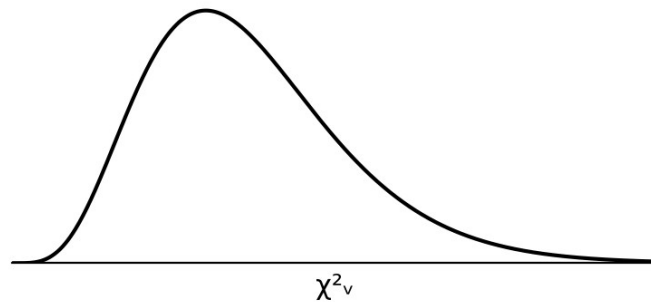


6A Variance of a Normal Distribution

Unbiased estimate of the population variance σ^2 : $s^2 =$

If $X_1, X_2, \dots, X_n$ is a random sample from $N(\mu, \sigma^2)$, the distribution of $(n - 1)S^2/\sigma^2$ is:

Using the table of percentage points of the χ^2 distribution:



100(1 - α)% confidence interval for the variance σ^2 of a normal distribution:

Confidence interval for the standard deviation σ :

1. In order to determine the accuracy of a new rifle, 8 marksmen were selected at random to fire the rifle at a target. The distances x , in mm, of the 8 shots from the centre of the target were as follows:

10, 14, 12, 8, 6, 11, 18, 14

Assuming that the distances are normally distributed, find a 95% confidence interval for the variance.

2. A company manufactures 12 amp electrical fuses. A random sample of 10 fuses was taken from a batch and the failure current (X) measured for each. The results are summarised below:

$$\Sigma x = 118.9 \quad \Sigma x^2 = 1414.89$$

Assume that the data can be regarded as a random sample from a normal population.

- a) Calculate an unbiased estimate for the variance of the batch based upon the sample.

- b) Use your estimate from part a to calculate a 95% confidence interval for the standard deviation.

6B Hypothesis Testing for the Variance of a Normal Distribution

Hypotheses for a test of the variance are written in terms of:

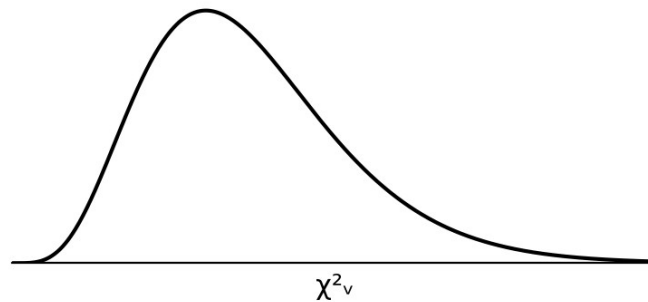
Test statistic and its distribution if H_0 is true:

The seven steps for a test of the variance of a normal distribution:

- 1
- 2
- 3
- 4
- 5
- 6
- 7

Two-tailed tests (the χ^2 distribution is not symmetrical):

1. A random sample of 12 observations is taken from a normal distribution with a variance of σ^2 . The unbiased estimate of the population variance is calculated as 0.015. Test, at the 5% level, the null hypothesis that $\sigma^2 = 0.025$ against the alternative hypothesis that $\sigma^2 \neq 0.025$.



6C The F-Distribution

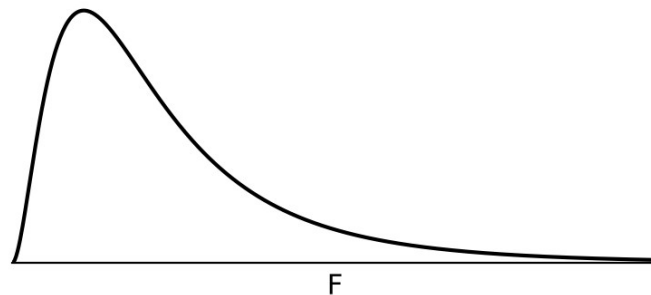
What the F-test is used for:

For independent random samples of sizes n_x and n_y from $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$:

If $\sigma_x^2 = \sigma_y^2$ this becomes:

Order of the degrees of freedom v_1 and v_2 :

Upper critical value $F_{v_1, v_2}(p)$:



Extract from the table of upper 5% critical values $F_{v_1, v_2}(0.05)$ (v_1 along the top, v_2 down the side):

$v_2 \backslash v_1$	1	2	3	4	5	6	8	10	12	24
1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	241.9	243.9	249.1
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.40	19.41	19.46
3	10.13	9.55	9.28	9.12	9.01	8.94	8.85	8.79	8.74	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.96	5.91	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.74	4.68	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.06	4.00	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.64	3.57	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.35	3.28	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.14	3.07	2.90

1. Use the table to find:
 - a) the $F_{5,8}(0.05)$ critical value
 - b) the $F_{8,5}(0.05)$ critical value.

Lower critical values: $F_{v_1, v_2}(1 - p) =$

2. Find upper critical values for:
 - a) $F_{8,10}(0.95)$
 - b) $F_{10,8}(0.95)$
3. Find the lower and upper 5% critical values for an $F_{a,\beta}$ -distribution in each of the following cases:
 - a) $a = 6, b = 10$
 - b) $a = 12, b = 8$
4. The random variable X follows an F-distribution with 8 and 10 degrees of freedom.
Find $P\left(\frac{1}{5.81} < X < 5.06\right)$

5. The random variable X follows an $F_{6,12}$ -distribution. Find $P(X < 0.25)$.

6D The F-Test

If $\sigma_x^2 = \sigma_y^2$, the value of s_x^2/s_y^2 should be close to:

The steps for an F-test of whether two variances are the same:

- 1
- 2
- 3
- 4
- 5
- 6
- 7

Two-tailed F-tests:

1. Two samples of sizes 13 and 9 are taken from normal distributions X and Y with variances σ_x^2 and σ_y^2 respectively. The two samples give values $s_x^2 = 24$ and $s_y^2 = 18$. Test, at the 5% level, $H_0: \sigma_x^2 = \sigma_y^2$ against $H_1: \sigma_x^2 > \sigma_y^2$.

2. Two samples of sizes 7 and 11 are taken from normal distributions X and Y with variances σ_x^2 and σ_y^2 respectively. The two samples give values $s_x^2 = 5$ and $s_y^2 = 25$. Test, at the 5% level of significance, $H_0: \sigma_x^2 = \sigma_y^2$ against $H_1: \sigma_x^2 < \sigma_y^2$.

3. Two samples of sizes 11 and 13 are taken from normal distributions X and Y with variances σ_x^2 and σ_y^2 respectively. The two samples give values $s_x^2 = 1.6$ and $s_y^2 = 2.4$. Test, at the 10% level of significance, $H_0: \sigma_x^2 = \sigma_y^2$ against $H_1: \sigma_x^2 \neq \sigma_y^2$.

4. A manufacturer of wooden furniture stores some of its wood outside and some inside a special store. It is believed that the wood stored inside should have less variable hardness properties than that stored outside. A random sample of 25 pieces of wood stored outside was taken and compared to a random sample of 21 similar pieces taken from the inside store, with the following results:

	Outside	Inside
Sample size	25	21
Mean hardness (coded units)	110	122
Sum of squares about the mean	5190	3972

- a) Test, at the 0.05 level of significance, whether the manufacturer's belief is correct.

- b) State an assumption you made in order to do this test.