

4A Combinations of Random Variables

Independent random variables:

If X and Y are two random variables, then

$$E(X + Y) = \quad \quad \quad E(X - Y) =$$

If X and Y are two independent random variables, then

$$\text{Var}(X + Y) = \quad \quad \quad \text{Var}(X - Y) =$$

1. If X is a random variable with $E(X) = \mu_1$ and $\text{Var}(X) = \sigma_1^2$ and Y is an independent random variable with $E(Y) = \mu_2$ and $\text{Var}(Y) = \sigma_2^2$, find the mean and variance of:

a) $X + Y$

b) $X - Y$

If X and Y are random variables: $E(aX \pm bY) =$

If X and Y are independent: $\text{Var}(aX \pm bY) =$

A linear combination of normally distributed random variables is

If $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ are independent, then

$$aX + bY \sim$$

$$aX - bY \sim$$

If $X_1, X_2, \dots, X_n$ are independent and identically distributed with $X_i \sim N(\mu, \sigma^2)$, then

$$X_1 + X_2 + \dots + X_n \sim$$

Watch out: $\text{Var}(X_1 + X_2) =$

but $\text{Var}(2X_1) =$

2. The independent random variables X and Y have distributions $X \sim N(5, 2^2)$ and $Y \sim N(10, 3^2)$.

a) Find the distribution of:

i) $A = X + Y$

ii) $B = 9X - 2Y$

b) Find $P(B > 30)$.

The independent random variables Y_1, Y_2, Y_3 and Y_4 all have the same distribution as Y . The random variable Z is defined as $Z = Y_1 + Y_2 + Y_3 + Y_4$.

c) Find the mean and standard deviation of Z .

3. The independent random variables X and Y have distributions $X \sim N(25, 6)$ and $Y \sim N(22, 10)$. Find $P(X > Y)$.

4. Bottles of mineral water are delivered to shops in crates containing 12 bottles each. The weights of bottles are normally distributed with mean weight 2 kg and standard deviation 0.05 kg. The weights of empty crates are normally distributed with mean 2.5 kg and standard deviation 0.3 kg.

a) Assuming that all random variables are independent, find the probability that a full crate will weigh between 26 kg and 27 kg.

b) Two bottles are selected at random from a crate. Find the probability that they differ in weight by more than 0.1 kg.

c) Find the weight, M , that a full crate should have on its label so that there is a 1% chance that it will weigh more than M .