

7A Mean of a Normal Distribution with Unknown Variance

For a random sample $X_1, X_2, \dots, X_n$ from a normal distribution with mean μ and unknown variance σ^2 :

Degrees of freedom of the t-distribution, and what happens as $v \rightarrow \infty$:

Shape of the t-distribution:

Percentage points of the t-distribution (the value of t exceeded with the probability shown):

v	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169

1. The random variable X has a t -distribution with 10 degrees of freedom. Determine values of t for which:

a) $P(X > t) = 0.025$

b) $P(X < t) = 0.95$

c) $P(X < t) = 0.025$

d) $P(|X| > t) = 0.05$

e) $P(|X| < t) = 0.98$

2. The random variable Y has a t_4 -distribution. Determine:

a) $P(Y > 3.747)$

b) $P(Y < -2.132)$

100(1 - α)% confidence interval for the mean μ of a normal distribution with unknown variance:

3. A sample of 6 trout taken from a fish farm were caught and their lengths in centimetres were measured. The lengths of the fish were as follows:

26.8 26.0 25.8 25.5 24.3 24.6

Assuming that the lengths of trout are normally distributed, find a 90% confidence interval for the mean length of trout in the fish farm.

4. The percentage starch content of potatoes is normally distributed with mean μ . In order to assess the mean value of the starch content, a random sample of 12 potatoes is selected and their starch content measured. The percentages of starch contents obtained were as follows:

23.2 20.3 18.6 20.0 20.8 21.6 19.4 18.7 22.1 19.5 21.3 22.6

Find a 95% confidence interval for the mean.