

5A Estimators, Bias and Standard Error

Random sample of size n:

Statistic:

1. A sample, $X_1, X_2, \dots, X_n$, is taken from a population with unknown population parameters μ and σ . State whether or not each of the following are statistics.

a $\frac{X_1 + X_3 + X_5}{3}$

b $\max(X_1, X_2, \dots, X_n)$

c $\sum_{i=1}^n \left(\frac{X_i - \mu}{\sigma} \right)^2$

Sampling distribution of a statistic T:

2. The weights, in grams, of a consignment of apples are normally distributed with a mean μ and standard deviation 4. A random sample of size 25 is taken and the statistics R and T are calculated as follows:

$$R = X_{25} - X_1 \quad \text{and} \quad T = X_1 + X_2 + \dots + X_{25}$$

Find the distributions of R and T.

3. A large bag contains counters. 60% of the counters have the number 0 on them and 40% have the number 1.
- a) Find the population mean μ and population variance σ^2 of the values shown on the counters.

x	0	1
P(X = x)		

A simple random sample of size 3 is taken from this population.

- b) List all the possible observations from this sample.

- c) Find the sampling distribution for the mean

$$\bar{X} = \frac{X_1 + X_2 + X_3}{3}$$

where X_1 , X_2 and X_3 are the values shown on the three counters in the sample.

$\bar{x}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1
P($\bar{X} = \bar{x}$)				

- d) Hence find $E(\bar{X})$ and $\text{Var}(\bar{X})$.

- e) Find the sampling distribution for the mode, M.

m	0	1
P(M = m)		

- f) Hence find $E(M)$ and $\text{Var}(M)$.

Estimator and estimate:

4. A random sample $X_1, X_2, \dots, X_n$ is taken from a population with $X \sim N(\mu, \sigma^2)$. Show that $E(\bar{X}) = \mu$.

Unbiased estimator:

Bias:

Unbiased estimator for σ^2 (the sample variance S^2) and ways to calculate s^2 :

5. A random sample $X_1, X_2, \dots, X_n$ is taken from a population with $X \sim N(\mu, \sigma^2)$.

$$\text{Show that } \text{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

Standard error:

Standard error of $\bar{X}$:

6. The table below summarises the number of breakdowns, X , on a town's bypass on 30 randomly chosen days.

Number of breakdowns	2	3	4	5	6	7	8	9
Number of days	3	5	4	3	5	4	4	2

- a) Calculate unbiased estimates of the mean and variance of the number of breakdowns.

20 more days were randomly sampled and this sample had $\bar{x} = 6.0$ and $s^2 = 5.0$.

- b) Treating the 50 results as a single sample, obtain further unbiased estimates of the population mean and variance.

- c) Find the standard error of this new estimate of the mean.

- d) Estimate the size of sample required to achieve a standard error of less than 0.25.