

5A The Central Limit Theorem

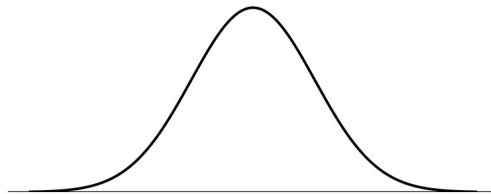
Sample mean of a normal population: if $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim$

The central limit theorem:

When is the distribution of $\bar{X}$ only an approximation? When does it get better?

Do you need a continuity correction?

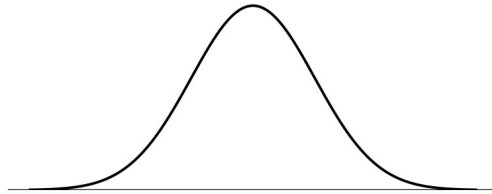
1. A sample of size 9 is taken from a population with distribution $N(10, 2^2)$. Find the probability that the sample mean $\bar{X}$ is more than 11.



2. A six-sided dice is relabelled so that there are three faces marked 1, two faces marked 3 and one face marked 6. The dice is rolled 40 times and the mean of the 40 scores is recorded.
 - a) Find an approximate distribution for the mean of the scores.

x			
P(X = x)			

b) Use your approximation to estimate the probability that the mean is greater than 3.



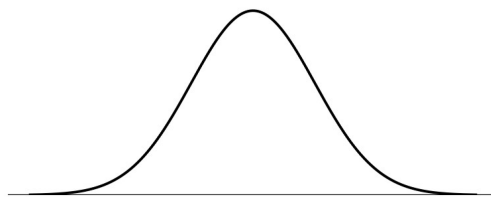
5B Applying the Central Limit Theorem to Other Distributions

Which distributions can you use the central limit theorem with?

Sum of observations and the sample mean:

Reminder: mean and variance of $Po(\lambda)$, $B(n, p)$, $Geo(p)$ and Negative $B(r, p)$:

1. A supermarket manager is trying to model the number of customers that visit her store each day. She observes that, on average, 20 new customers enter the store every minute.
 - a) Calculate the probability that fewer than 15 customers arrive in a given minute.
 - b) Find the probability that in one hour no more than 1150 customers arrive.
 - c) Use the central limit theorem to estimate the probability that in one hour no more than 1150 customers arrive. Compare your answer to part b.



2. Billy is the captain of a football team. Each week he gets a team together by calling his friends one by one and asking if they would like to play. The probability of each friend agreeing to play is $\frac{2}{3}$. Once he has 10 other players he stops calling.
- a) Calculate the number of friends Billy expects to have to call to find 10 other players.

- b) Find the probability that Billy has to call exactly 12 friends.

In a season, Billy's team plays 25 matches.

- c) Estimate the probability that the mean number of calls per match Billy had to make was less than 15.5.

