

4A Testing for the Mean of a Poisson Distribution

When can a Poisson distribution be used as a model?

Null hypothesis, H_0 :

Alternative hypothesis, H_1 :

One-tailed test:

Two-tailed test:

Steps for a hypothesis test for the mean λ of a Poisson distribution:

1. Accidents used to occur at a certain road junction at the rate of 6 per month. The residents petitioned for traffic lights. In the month after the lights were installed there was only one accident. Test, at the 5% level of significance, whether there is evidence that the lights have reduced the rate of accidents.
2. Over a long period of time, Fatima found that the bus taking her to school was late at a rate of 2.5 times per month. In the month following the start of the new summer bus schedules, Fatima finds that her bus is late 6 times. Assuming that the number of times the bus is late each month has a Poisson distribution, test at the 2% level of significance, whether or not the new schedules changed the frequency with which the bus is late.

Using a Poisson approximation to the binomial distribution $B(n, p)$:

3. During an influenza epidemic, 4% of a population of a large city were affected on a given day. The manager of a factory that employs 250 people found that 17 of the employees at his factory were absent, claiming to be suffering from influenza. Using a Poisson approximation to the binomial distribution and a 5% level of significance, test whether or not the proportion of employees suffering from influenza at his factory was larger than that of the whole city.

4B Finding Critical Regions for a Poisson Distribution

Critical region:

Critical value:

One-tailed test ($H_1: \lambda < m$ or $H_1: \lambda > m$):

Two-tailed test ($H_1: \lambda \neq m$):

Actual significance level:

1. An estate agent has been selling houses at a rate of 9 per month. He believes that the rate of sales will decrease in the next month.
 - a) Using a 5% level of significance, find the critical region for a one-tailed test of the hypothesis that the rate of sales will decrease from 9 per month.

x	2	3	4	5
P(X ≤ x)	0.0062			

- b) Write down the actual significance level of the test in part a.

2. Leonora Metti is regarded as a super striker who plays for Statistics All Stars. The mean number of goals she has scored over the several years she has been with the club is 0.7 goals per game. She has now been transferred to Mechanics Ladies. The management at Mechanics Ladies wish to see if the rate at which she scores goals has now increased. They monitor the number of goals that she scores in her first 10 games.

Assuming a Poisson distribution,

- a) use a 5% level of significance to find the critical region for a one-tailed test of the hypothesis that Leonora has increased her rate of scoring from 0.7 goals per game

x	10	11	12	13
P(X ≤ x)	0.9015			
P(X ≥ x + 1)	0.0985			

b) write down the actual significance level of the test in part a.

Leonora scores 11 goals in her first 10 games for Mechanics Ladies.

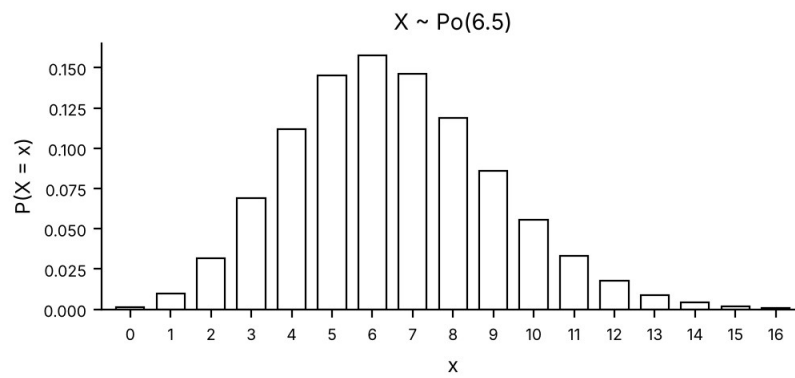
c) Comment on this observation in light of your critical region.

3. An office finds that, over a long period of time, incoming calls from customers occur at a rate of 0.325 per minute. They believe that the rate of calls has changed recently. To test this, the number of incoming calls during a random 20-minute interval is recorded.

a) Find the critical region for a two-tailed test of this hypothesis. The probability in each tail should be as close to 2.5% as possible.

x	0	1	2	3	10	11	12	13
P(X ≤ x)	0.0015				0.9332			

Shade the critical region on the diagram:



b) Write down the actual significance level of the test.

The actual number of calls recorded in the 20-minute period was 13.

c) Comment on this observation in light of your critical region.

4C Hypothesis Testing for the Parameter p of a Geometric Distribution

Conditions for a geometric distribution $X \sim \text{Geo}(p)$ to be valid:

Mean of $X \sim \text{Geo}(p)$:

Null hypothesis, H_0 :

Alternative hypothesis, H_1 :

What happens to the mean of $\text{Geo}(p)$ when p increases? Which end of the distribution do you look at?

Useful results: if $X \sim \text{Geo}(p)$,

$$P(X = x) =$$

$$P(X \leq x) =$$

$$P(X \geq x) =$$

1. A company claims that 1 in 8 of their packets of crisps contains a prize ticket. Lee decides to test this claim and he buys a packet of crisps each day until he finds a prize ticket. Lee finds a prize ticket for the first time on the 24th day. Test, at the 5% significance level, whether there is any evidence to suggest that the company is overstating the proportion of packets containing a prize ticket.

2. An electronics company makes small components for use in computers. It claims that the percentage of defective components coming off the production line is 0.05%. The electronics company sells the components to a retailer. The retailer suspects that the percentage defective stated by the electronics company should be higher. From a very large consignment recently purchased, the retailer tests the components until he finds a defective component. He finds a defective component on the 90th component that he tests. Is there any evidence to suggest that the retailer's suspicions are correct? Test at the 5% level of significance.

4D Finding Critical Regions for a Geometric Distribution

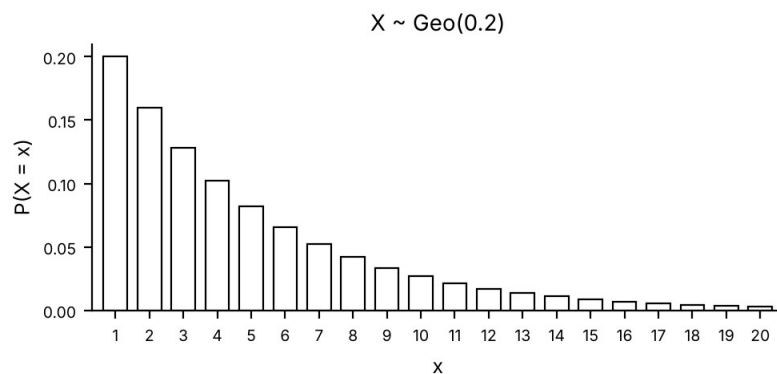
Testing $H_1: p < m$: the critical region is at which end of the distribution? Which probability do you use?

Testing $H_1: p > m$: the critical region is at which end of the distribution? Which probability do you use?

Solving an inequality such as $0.8^{c-1} < 0.05$ using logs (watch out for the direction of the inequality):

1. Ayesha wishes to test if a five-sided spinner numbered from 1 to 5 is fair. She spins the spinner and counts the number of trials until she gets the spinner to show the number 1.
 - a) Using a 5% level of significance, find the critical region for a one-tailed test of the hypothesis that the probability of getting the number 1 on a single spin is less than $\frac{1}{5}$.

Shade the critical region on the diagram:



- b) Write down the actual significance level of the test in part a.

2. In a particular city, a Lobster Card is used as a method of payment on trains. The company that administers the card claims that only 1 in 1000 cards will be rejected by the card reader at the train station. A station manager feels that the company is understating the proportion of cards rejected by the card reader, and decides to carry out a test. When he comes on shift at 5:00am he counts the number of passengers who pass through until he notes a passenger who has a Lobster Card rejected.
 - a) Using a 5% level of significance, find the critical region for a one-tailed test of the hypothesis that the proportion of Lobster Cards rejected by the card reader is greater than 1 in 1000.
 - b) Write down the actual significance level of the test in part a.