

1A Expected Value of a Discrete Random Variable

Discrete random variable:

Expected value of X , $E(X)$:

The probabilities of a discrete random variable:

1. A fair six-sided dice is rolled. The number on the uppermost face is modelled by the random variable X .

a) Write down the probability distribution of X .

x	1	2	3	4	5	6
$P(X = x)$						

b) Use the probability distribution of X to calculate $E(X)$.

Does the expected value have to be a value X can take?

2. The random variable X has a probability distribution as shown in the table.

x	1	2	3	4	5
$P(X = x)$	0.1	p	0.3	q	0.2

a) Given that $E(X) = 3$, write down two equations involving p and q .

b) Find the value of p and the value of q .

Expected value of X^2 , $E(X^2)$:

3. A discrete random variable X has the following probability distribution

x	1	2	3	4
$P(X = x)$	12/25	6/25	4/25	3/25

- a) Write down the probability distribution for X^2 .

x	1	2	3	4
x^2				
$P(X^2 = x^2)$				

- b) Find $E(X^2)$.

Is $E(X^2)$ equal to $(E(X))^2$?

1B Variance of a Discrete Random Variable

Variance of X , $\text{Var}(X)$:

A quicker formula for $\text{Var}(X)$:

Notation (σ^2 and σ):

What does a large variance tell you?

1. A fair six-sided dice is rolled. The number on the uppermost face is modelled by the random variable X . Find $\text{Var}(X)$.

Method 1: using $\text{Var}(X) = E((X - E(X))^2)$

x	1	2	3	4	5	6
x^2						
$(x - E(X))^2$						
$P(X = x)$						

Method 2: using $\text{Var}(X) = E(X^2) - (E(X))^2$

1C Expected Value and Variance of a Function of X

Expected value of a function, $E(g(X))$:

$E(aX + b)$:

$E(X + Y)$:

$\text{Var}(aX + b)$:

1. A discrete random variable X has the probability distribution

x	1	2	3	4
$P(X = x)$	12/25	6/25	4/25	3/25

- a) Write down the probability distribution for Y where $Y = 2X + 1$.

x	1	2	3	4
y				
$P(Y = y)$				

- b) Find $E(Y)$.

- c) Compute $E(X)$ and verify that $E(Y) = 2E(X) + 1$.

2. A random variable X has $E(X) = 4$ and $\text{Var}(X) = 3$. Find:

- a) $E(3X)$

- b) $E(X - 2)$

- c) $\text{Var}(3X)$

d) $\text{Var}(X - 2)$

e) $E(X^2)$

3. Two fair 10p coins are spun. The random variable X pence represents the total value of the coins that land heads up.

- a) Find $E(X)$ and $\text{Var}(X)$.

x			
P(X = x)			

The random variables S and T are defined as follows: $S = X - 10$ and $T = \frac{1}{2}X - 5$

- b) Show that $E(S) = E(T)$.

- c) Find $\text{Var}(S)$ and $\text{Var}(T)$.

A large number of observations of S and T are taken.

- d) Comment on any likely differences or similarities.

4. The random variable X has the following probability distribution. Calculate $E(\sin X)$.

x	0°	30°	60°	90°
P(X = x)	0.4	0.2	0.1	0.3

sin x				
P(X = x)	0.4	0.2	0.1	0.3

1D Solving Problems Involving Random Variables

If $Y = g(X)$ and g is one-to-one, how can you find $E(X)$ and $\text{Var}(X)$ from $E(Y)$ and $\text{Var}(Y)$?

1. X is a discrete random variable. The discrete random variable Y is defined as $Y = (X - 150)/50$. Given that $E(Y) = 5.1$ and $\text{Var}(Y) = 2.5$, find:
- a) $E(X)$

b) $\text{Var}(X)$.

2. The discrete random variable X has a probability distribution given by

x	-2	-1	0	1	2
P(X = x)	0.3	a	0.25	b	c

The discrete random variable Y is defined as $Y = 3X - 1$. Given that $E(Y) = -2.5$ and $\text{Var}(Y) = 13.95$, find:

a) $E(X)$ and $E(X^2)$

b) the values of a , b and c

c) $P(X > Y)$