

## 8D The Power Function

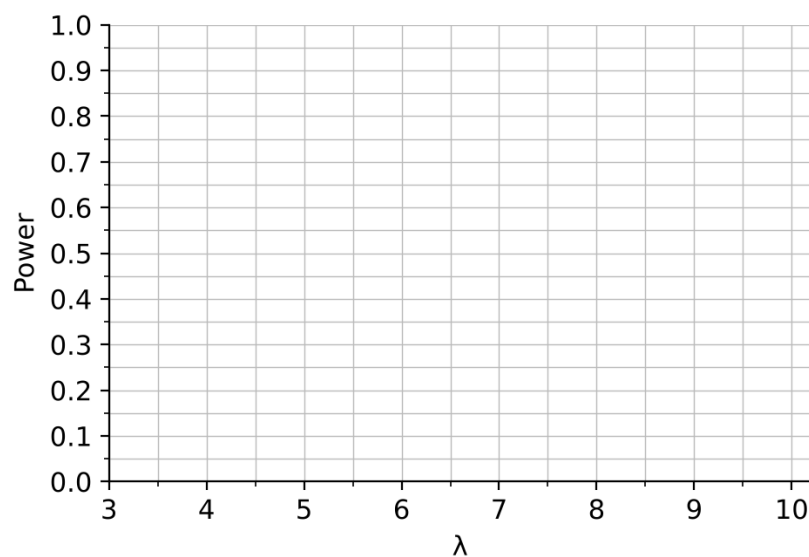
Power function:

1. Past experience has shown that the number of accidents that take place at a road junction has a Poisson distribution with an average of 3.5 accidents per month. A trading estate is built along one of the roads leading away from the junction and the local council is anxious that this may have increased the accident rate. To see if the number of accidents had increased, a test was set up with the null hypothesis  $H_0: \lambda = 3.5$  and with the alternative hypothesis being accepted if the number of accidents  $X$  within the first month after the alteration was  $\geq 7$ .

a) Find the size of the test.

b) Find the power function for the test and sketch the graph of the power function.

| $\lambda$ | 4      | 5      | 6 | 7 | 8 | 9 | 10 |
|-----------|--------|--------|---|---|---|---|----|
| Power     | 0.1107 | 0.2378 |   |   |   |   |    |



Comparing two tests:

2. A manufacturer of sweets supplies a mixed assortment of chocolates in a jar. He claims that 40% of the chocolates have a 'hard centre', the remainder being 'soft centred'.

A shopkeeper does not believe the manufacturer's claim and proposes to test it using the hypotheses  $H_0: p = 0.4$ ,  $H_1: p < 0.4$ , where  $p$  is the proportion of 'hard centres' in the jar. Two tests are proposed.

In test A he takes a random sample of 10 chocolates from the jar and rejects  $H_0$  if the number of 'hard centres' is less than 2.

a) Find the size of test A.

b) Show that the power function of test A is given by  $(1 - p)^{10} + 10p(1 - p)^9$ .

In test B he takes a random sample of 5 chocolates from the jar and if there are no 'hard centres' he rejects  $H_0$ , otherwise he takes a second sample of 5 chocolates and  $H_0$  is rejected if there are no further 'hard centres' on this second occasion.

c) Find the size of test B.

d) Find an expression for the power function of test B.

The powers for test A and test B for various values of  $p$  are given in the table.

| <b>p</b>                | <b>0.1</b> | <b>0.2</b> | <b>0.25</b> | <b>0.3</b> | <b>0.35</b> |
|-------------------------|------------|------------|-------------|------------|-------------|
| <b>Power for test A</b> | 0.74       | r          | 0.24        | s          | 0.09        |
| <b>Power for test B</b> | 0.83       | 0.54       | 0.42        | 0.31       | 0.22        |

e) Calculate values for r and s.

- f) State, giving a reason, which of the two tests the shopkeeper should use.
3. A local park believes the fox population in the area has decreased. They want to test for the probability,  $p$ , that a fox will be observed on any given day. They count the number of days,  $X$ , that pass until the first observation of a fox. They test  $H_0: p = 0.1$  against  $H_1: p < 0.1$  and reject  $H_0$  if  $X > 30$ .
- a) Find the size of this test.
- b) Find the power function for the test.