

8A Type I and Type II Errors

Critical region (discrete distributions):

Actual significance level:

1. One rainy day during the summer holidays, a family of four were playing a simple game of cards. The game was one of chance so the probability of any particular person winning should have been $\frac{1}{4}$. After playing a number of games, Robert complained that his younger sister Sarah must have been cheating as she kept winning. Their parents quickly intervened and decided to carry out a proper investigation and carefully watched the next 20 games.
Find the critical region for a one-tailed test using a 5% level of significance.

Type I error:

Type II error:

Complete the table to show the types of error that can occur in a hypothesis test.

Test result	H_0 is true	H_0 is false
Accept H_0		
Reject H_0		

2. Use the situation in question 1.
 - a) Find the probability of a Type I error.
 - b) If in fact Sarah was cheating and $p = 0.35$, find the probability of a Type II error.
3. Accidents occurred on a stretch of motorway at an average rate of 6 per month. Many of the accidents that occurred involved vehicles skidding into the back of other vehicles. By way of a trial, a new type of road surface that is said to reduce the risk of vehicles skidding is laid on this stretch of road, and during the first month of operation 4 accidents occurred.
 - a) Test this result to see if it gives evidence that there has been an improvement at the 5% level of significance.
 - b) Calculate $P(\text{Type I error})$ for this test.
 - c) If the true average rate of accidents occurring with the new type of road surface was 3.5, calculate the probability of a Type II error.

4. A coin is spun 20 times and a head is obtained on 7 occasions.
- a) Test to see whether or not the coin is biased.
 - b) Calculate the probability of a Type I error for this test.
 - c) Given that the coin is biased and that this bias causes the tail to appear 3 times for each head that appears, calculate the probability of a Type II error for the test.

For $X \sim \text{Geo}(p)$: $P(X \leq n) =$

$P(X \geq n) =$

5. Jane knows from experience that 10% of the emails she receives are spam. After her email service upgraded the spam filters, she recorded the number of emails sent up to and including the first spam email. She wants to test, at the 5% significance level, whether this upgrade improved the spam filter.
- a) Find the critical region for her test.

b) Calculate the probability of a Type I error for this test.

c) Given that after the upgrade the probability of an email she receives being spam is now 1 in a 100, calculate the probability of a Type II error for the test.