

7B Probability Generating Functions of Standard Distributions

The probability generating functions of the standard distributions (given in the formulae booklet):

If $X \sim B(n, p)$: $G_X(t) =$

If $X \sim \text{Po}(\lambda)$: $G_X(t) =$

If $X \sim \text{Geo}(p)$: $G_X(t) =$

If $X \sim \text{Negative B}(r, p)$: $G_X(t) =$

'Write down' or 'find' the p.g.f. of a standard distribution means:

'Show from first principles' means:

Useful results: Maclaurin series $e^x =$

Sum to infinity of a geometric series $S_\infty =$

3. An archer hits the bullseye with probability 0.6. She fires three shots at a target. The random variable X represents the number of times she hits the bullseye. Find the probability generating function of X .

x	0	1	2	3
$P(X = x)$				

4. X is a discrete random variable such that $X \sim \text{Po}(1.1)$. Show, from first principles, that the probability generating function of X is given by $G_X(t) = e^{1.1(t-1)}$

5. A fair tetrahedral dice is rolled until the dice shows a four. The discrete random variable X represents the number of rolls up to and including the first instance of a four. Show, from first principles, that the probability generating function for X is given by

$$G_X(t) = \frac{\frac{1}{4}t}{1 - \frac{3}{4}t}$$

6. The discrete random variable $X \sim \text{Negative B}(r, p)$. Prove, from first principles, that the probability generating function of X can be written as

$$G_X(t) = \left(\frac{pt}{1 - (1 - p)t} \right)^r$$

You may quote the following result without proof:

$$\sum_{x=r}^{\infty} \binom{x-1}{r-1} q^{x-r} = (1-q)^{-r} \quad \text{where } q = 1 - p$$