

3A Loci in an Argand Diagram

The locus of z where $|z - z_1| = r$ is:

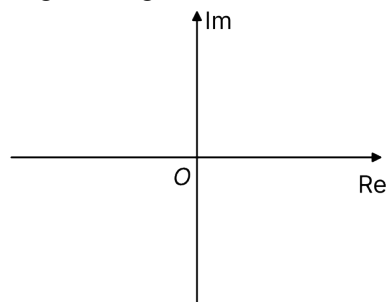
The locus of z where $|z - z_1| = |z - z_2|$ is:

The locus of z where $|z - a| = k|z - b|$, with $k > 0$ and $k \neq 1$, is:

How to find its centre and radius:

1. Given that $|z - 6| = 2|z + 6 - 9i|$,
 - a) use algebra to show that the locus of z is a circle, stating its centre and its radius

- b) sketch the locus of z on an Argand diagram.



The locus of z where $\arg(z - z_1) = \theta$ is:

Three circle theorems that help with arg loci:

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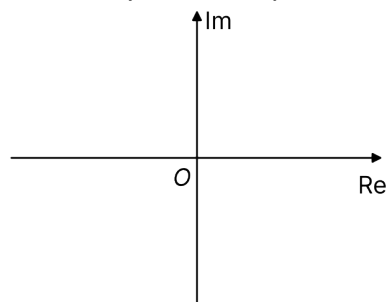
The locus of z where $\arg\left(\frac{z-a}{z-b}\right) = \theta$, $\theta > 0$, is:

Why (use $\arg z_1 - \arg z_2 = \arg(z_1/z_2)$):

	Shape of the locus
$\theta < \pi/2$	
$\theta = \pi/2$	
$\theta > \pi/2$	

Which way round the arc goes, and what happens at the endpoints A and B:

2. Given that $\arg\left(\frac{z-6}{z-2}\right) = \frac{\pi}{4}$,
- a) sketch the locus of $P(x, y)$ which is represented by z on an Argand diagram

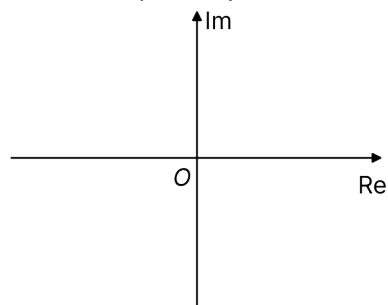


- b) find the Cartesian equation of this locus.

Method 1 (geometric):

Method 2 (algebraic):

3. Given the equation $\arg\left(\frac{z}{z-4i}\right) = \frac{\pi}{2}$,
- a) sketch the locus of points z that satisfy the equation on an Argand diagram



- b) Hence write down the range of possible values of $\text{Re}(z)$.

3B Regions in an Argand Diagram

The region $\theta_1 \leq \arg(z - z_1) \leq \theta_2$ is:

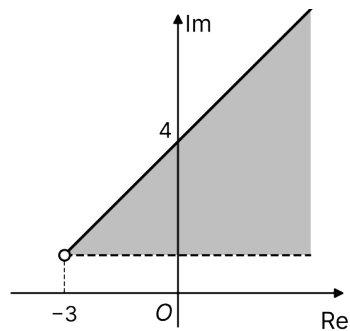
Solid and dashed boundary lines ($\leq$ and $<$):

The region $|z - z_1| \leq r$ is:

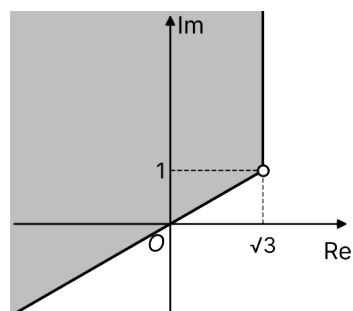
The symbols $\cap$ and $\cup$ mean:

1. Describe algebraically, in terms of z , the region shown in each Argand diagram.

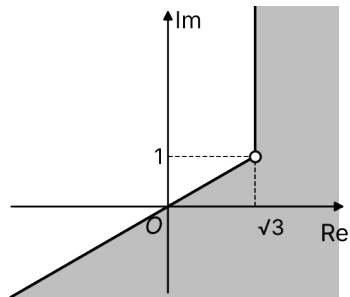
a)



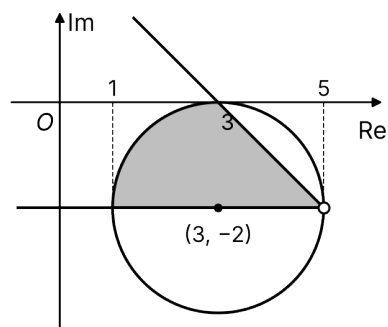
b)



c)



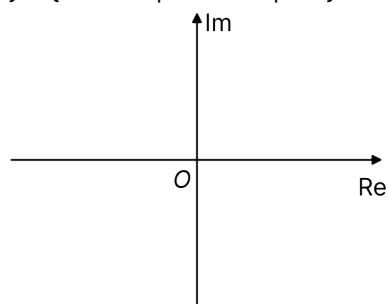
d)



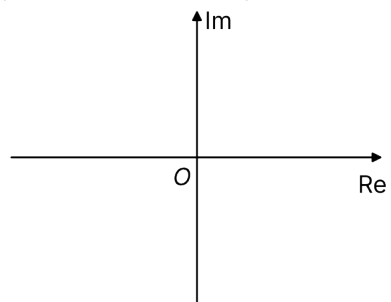
Tip for sketching an intersection of regions:

2. On separate Argand diagrams, shade the region satisfied by each set of points:

a) $\{z \in \mathbb{C} : \frac{2\pi}{3} \leq \arg z \leq \pi\} \cap \{z \in \mathbb{C} : |z + 3 - 4i| \leq 5\}$



b) $\{z \in \mathbb{C} : 2|z - 4| \leq |z|\} \cap \{z \in \mathbb{C} : 4 \leq \operatorname{Re}(z) \leq 6\}$



3C Transformations of the Complex Plane

A transformation maps the z -plane ($z = x + iy$) to the w -plane ($w = u + iv$). Method for finding the image of a locus:

	Transformation it represents
$w = z + a + ib$	
$w = kz$ (k real)	
$w = iz$	
$w = kz + a + ib$	

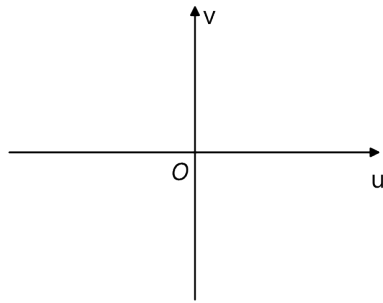
1. The point P represents the complex number z on an Argand diagram, where $|z| = 2$. T_1 , T_2 and T_3 represent transformations from the z -plane, where $z = x + iy$, to the w -plane, where $w = u + iv$. Describe the locus of the image of P under the transformations:

a) $T_1: w = z - 2 + 4i$

b) $T_2: w = 3z$

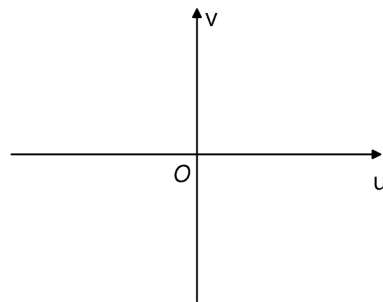
c) $T_3: w = \frac{1}{2}z + i$

2. For the transformation $w = iz - 1$, find the locus of w when z lies on the half-line $\arg(z + 2) = \frac{\pi}{4}$.



3. A transformation from the z -plane to the w -plane is given by $w = z^2$, where $z = x + iy$ and $w = u + iv$. Describe the locus of w and give its Cartesian equation when z lies on:
- a) a circle with equation $x^2 + y^2 = 16$

- b) the line with equation $x = 1$

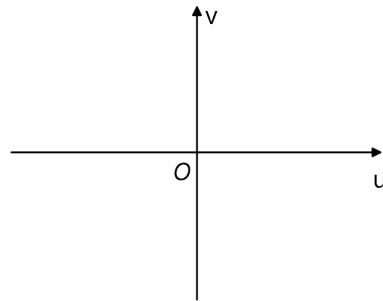


Transformations of the form $w = \frac{az + b}{cz + d}$, $a, b, c, d \in \mathbb{C}$, are called:

Method:

4. The transformation T from the z -plane, where $z = x + iy$, to the w -plane, where $w = u + iv$, is given by $w = \frac{5iz + i}{z + 1}$, $z \neq -1$.
- a) Show that the image, under T , of the circle $|z| = 1$ in the z -plane is a line l in the w -plane.

b) Sketch I on an Argand diagram.



5. The transformation T from the z -plane, where $z = x + iy$, to the w -plane, where $w = u + iv$, is given by $w = \frac{3z - 2}{z + 1}$, $z \neq -1$. Show that the image, under T , of the circle with equation $x^2 + y^2 = 4$ in the z -plane is a circle C in the w -plane. State the centre and radius of C .

6. A transformation T of the z -plane to the w -plane is given by $w = \frac{iz - 2}{1 - z}$, $z \neq 1$. Show that as z lies on the real axis in the z -plane, then w lies on a line l in the w -plane. Sketch l on an Argand diagram.

