

6C Area of a Surface of Revolution

A surface of revolution is:

Area of the thin band made by rotating a small arc δs , a distance y from the x -axis:

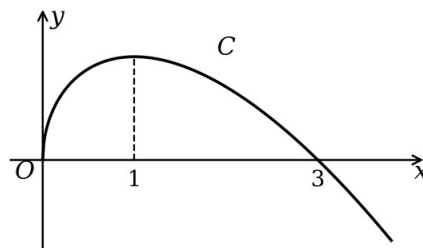
Surface area for $y = f(x)$ rotated through 2π about the x -axis:

Surface area for a Cartesian curve rotated through 2π about the y -axis (two forms):

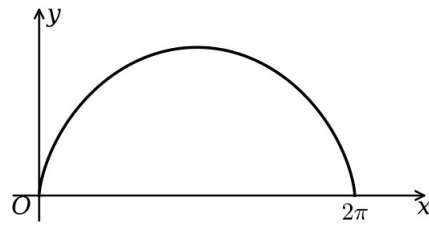
Surface area for a parametric curve $x = f(t)$, $y = g(t)$, rotated about the x -axis and about the y -axis:

Surface area for a polar curve $r = f(\theta)$, rotated about the initial line and about the line $\theta = \pi/2$:

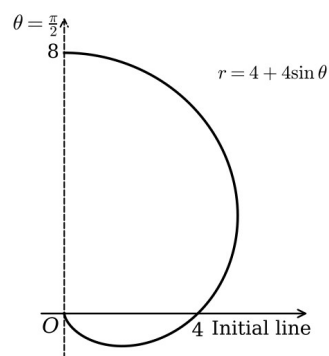
1. The curve C has equation $y = \frac{1}{3}\sqrt{x}(3 - x)$. The arc of the curve between the points with x -coordinates 1 and 3 is completely rotated about the x -axis. Find the area of the surface generated.



2. The curve with parametric equations $x = t - \sin t$, $y = 1 - \cos t$, from $t = 0$ to $t = 2\pi$, is rotated through 360° about the x-axis. Find the area of the surface generated.



3. The graph of the polar equation $r = 4 + 4\sin \theta$ from $\theta = -\pi/2$ to $\theta = \pi/2$ is shown below. The curve is rotated through 2π radians about the vertical line $\theta = \pi/2$, forming a surface of revolution. Find the exact area of the surface.



4. The arc of the curve with equation $y = \cosh x$, from the point $(0, 1)$ to $(\ln 2, 5/4)$, is rotated completely about the y -axis. Find the area of the surface generated.

