

5A/B Eigenvalues and Eigenvectors

An eigenvector of a matrix A is:

The eigenvalue corresponding to the eigenvector x is:

Under the transformation represented by A, what happens to an eigenvector? What does the eigenvalue tell you?

Eigenvectors and invariant lines:

If the corresponding eigenvalue is 1:

Why $\det(A - \lambda I) = 0$ gives the eigenvalues:

The characteristic equation of A:

1. Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 5 \\ -1 & -4 \end{pmatrix}$$

A normalised eigenvector:

If $\mathbf{a} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is an eigenvector, then $\hat{\mathbf{a}} =$

Normalised eigenvectors for question 1:

Repeated eigenvalues and complex eigenvalues:

Linearly independent vectors:

2. Find the eigenvalues and corresponding eigenvectors for these matrices:

a)

$$\mathbf{A} = \begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix}$$

b)

$$\mathbf{B} = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$$

c)

$$\mathbf{C} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$$

3. A transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is represented by the matrix

$$\mathbf{A} = \begin{pmatrix} 4 & -5 \\ 1 & -2 \end{pmatrix}$$

a) Find the eigenvalues of $\mathbf{A}$.

b) Find Cartesian equations of the two lines passing through the origin which are invariant under T .

Eigenvalues and eigenvectors of a 3×3 matrix (what kind of equation? how do you factorise it?):

Why a 3×3 matrix always has at least one real eigenvalue:

4. Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & -3 \\ 0 & 2 & 1 \\ 0 & -4 & -3 \end{pmatrix}$$

5. The matrix

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 5 \\ 2 & 1 & 0 \end{pmatrix}$$

- a) Show that -2 is the only real eigenvalue of $\mathbf{A}$.

- b) Find a normalised eigenvector of A corresponding to the eigenvalue -2 .

6. The matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

- a) Show that 1 is a repeated eigenvalue of A and find the other distinct eigenvalue.

- b) Find two linearly independent eigenvectors corresponding to the eigenvalue 1 .