4C Solving Second-Order Recurrence Relations

A second-order linear recurrence relation can be written in the form:

1. Consider the recurrence relation $u_n = 2u_{n-1} - u_{n-2}$, $n \geq 2$. Verify that the following particular solutions satisfy this recurrence relation.
   a) $u_n = 3n$

   b) $u_n = 5$

   c) $u_n = 3n + 5$

If $u_n = F(n)$ and $u_n = G(n)$ are both solutions, then:

The auxiliary equation of $u_n = au_{n-1} + bu_{n-2}$, and where it comes from:

	Form of the general solution
Distinct real roots α, β	
Repeated root α	
Complex roots $re^{\pm i\theta}$	

2.
   a) Find a general solution to the recurrence relation $u_n = 2u_{n-1} + 8u_{n-2}$, $n \geq 2$.

   b) Verify that your general solution from part a satisfies the recurrence relation.

c) Given that $u_0 = 4$ and $u_1 = 10$, find a particular solution.

3. Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$, with $a_1 = 5$ and $a_2 = 3$.

4. Solve the recurrence relation $u_n = 4u_{n-1} - 4u_{n-2}$, with $u_0 = 1$ and $u_1 = 1$.

5.

a) Find the general solution to the recurrence relation $u_n = 2u_{n-1} - 2u_{n-2}$.

b) Given that $u_0 = 1$ and $u_2 = 2$, find the particular solution to the recurrence relation.

6. The Fibonacci sequence is defined recursively as $F_n = F_{n-1} + F_{n-2}$, $n > 2$, with $F_1 = 1$ and $F_2 = 1$. Find a closed form for F_n.

Solving the non-homogeneous recurrence relation $u_n = au_{n-1} + bu_{n-2} + g(n)$:

	Form of particular solution
p with $\alpha, \beta \neq 1$	
pn + q with $\alpha, \beta \neq 1$	
kp^n with $p \neq \alpha, \beta$	
p with $\alpha = 1, \beta \neq 1$	
pn + q with $\alpha = 1, \beta \neq 1$	
p with $\alpha = \beta = 1$	
pn + q with $\alpha = \beta = 1$	
$k\alpha^n$ with $\alpha \neq \beta$	
$k\alpha^n$ with $\alpha = \beta$	

7. Solve the recurrence relation $a_{n+2} + 4a_{n+1} + 3a_n = 5(-2)^n$, $n \geq 0$, with $a_0 = 2$ and $a_1 = -1$.

8. Find the general solution to $s_n = 3s_{n-1} + 4s_{n-2} + 4^n$.