

2A The Axioms for a Group

Binary operation:

A set S is closed under a binary operation $*$ if:

1. $S = \{x + y\sqrt{3} : x, y \in \mathbb{Z}\}$. Show that addition is a binary operation on S .
2. Show that the set of natural numbers, $\mathbb{N} = \{1, 2, 3, 4, \dots\}$, is not closed under subtraction.

Identity element of a set S under a binary operation $*$:

3. Prove that an identity element of a set S under a binary operation must be unique.

Inverse elements (a and b are inverses of each other if...):

4. The binary operation $*$ on the set of real numbers is defined as $a * b = a + b + ab$.
 - a) Find a real number e that satisfies the property $a * e = a$.

The real number m has inverse m^{-1} that satisfies the property $m * m^{-1} = e$.

- b) Express m^{-1} in terms of m .

A binary operation $*$ on a set S is associative if:

5. A binary operation on $\mathbb{R}$ is defined by $a \circ b = ab + 1$. Show that $\circ$ is not associative.

If G is a set and $*$ is a binary operation defined on G , then $(G, *)$ is a group if these four axioms hold:

- Closure:
- Identity:
- Inverses:
- Associativity:

6. Show that:
- a) the set of integers forms a group under addition

- b) the set of integers does not form a group under multiplication.

7. The operation $*$ is defined by $a * b = a + b - 1$, where a and b are real numbers. Determine whether the set of real numbers under the operation $*$ forms a group.

8. Prove that, for all elements a, b in a group $(G, *)$, there exists a unique element c such that $a * c = b$.

The latin square property: