

5A Motion in a Straight Line with Variable Force

When the force is a function of time, $F = f(t)$, use $F = ma$ with $a =$

When the force is a function of the displacement, $F = f(x)$, use $F = ma$ with $a =$

When the force is a function of the velocity, use $a = dv/dt$ (time involved) or $a = v dv/dx$ (distance involved). Watch out for:

1. A particle P of mass 0.5 kg is moving along the x-axis. At time t seconds the force acting on P has magnitude $(5t^2 + e^{0.2t})$ N and acts in the direction OP. When $t = 0$, P is at rest at O. Calculate:
 - a) the speed of P when $t = 2$

- b) the distance OP when $t = 3$.

2. A pebble of mass 0.2 kg is moving on a smooth horizontal sheet of ice. At time t seconds (where $t \geq 0$) a horizontal force of magnitude $2t^2 \text{ N}$ and constant direction acts on the pebble. When $t = 0$ the pebble is moving in the same direction as the force and has speed 6 m s^{-1} . When $t = T$ the pebble has speed 36 m s^{-1} . Calculate the value of T .
3. A particle P of mass 1.5 kg is moving in a straight line. The force acting on P has magnitude $(8 - 2 \cos x) \text{ N}$, where x metres is the distance OP , and acts in the direction OP . When P passes through O its speed is 4 m s^{-1} . Calculate the speed of P when $x = 2$.

Work done by a variable force $F(x)$ moving a particle from $x = a$ to $x = b$:

4. A stone S of mass 0.5 kg is moving in a straight line on a smooth horizontal floor. When S is a distance x metres from a fixed point on the line, A , a force of magnitude $(5 + 7 \cos x) \text{ N}$ acts on S in the direction AS . Given that S passes through A with speed 2 m s^{-1} , calculate:
- a) the speed of S as it passes through the point B , where $x = 3$

b) the work done by the force in moving S from A to B.

5. A particle P of mass 0.5 kg moves in a straight horizontal line. When the speed of P is $v \text{ m s}^{-1}$, the resultant force acting on P is a resistance of magnitude $3v \text{ N}$. Find the distance moved by P as it slows down from 12 m s^{-1} to 6 m s^{-1} .

6. A particle P of mass m is moving along Ox in the direction of x increasing. At time t , the only force acting on P is a resistance of magnitude $mk(c^2 + v^2)$, where v is the speed of P and c and k are positive constants. When $t = 0$, P is at O and $v = U$. The particle P comes to rest at the point A. Find:
- a) the distance OA

b) the time P takes to travel from O to A.

7. A car of mass 800 kg travels along a straight horizontal road. The engine of the car produces a constant driving force of magnitude 2000 N. At time t seconds, the speed of the car is $v \text{ m s}^{-1}$. As the car moves, the total resistance to the motion of the car is of magnitude $(400 + 4v^2) \text{ N}$. The car starts from rest.

a) Find v in terms of t .

b) Show that the speed of the car cannot exceed 20 m s^{-1} .

5B Newton's Law of Gravitation

Newton's law of gravitation:

G is the constant of gravitation. Its value is:

The inverse square law is used when:

8. Two particles of masses 0.5 kg and 2.5 kg are 4 cm apart. Calculate the magnitude of the gravitational force between them.

Relationship between G and g (model the Earth as a sphere of mass M and radius R):

9. Above the Earth's surface, the magnitude of the force on a particle due to the Earth's gravitational force is inversely proportional to the square of the distance of the particle from the centre of the Earth. The acceleration due to gravity on the surface of the Earth is g and the Earth can be modelled as a sphere of radius R . A particle P of mass m is a distance $(x - R)$, (where $x \geq R$), above the surface of the Earth.
 - a) Prove that the magnitude of the gravitational force acting on P is mgR^2/x^2 .

A spacecraft S is fired vertically upwards from the surface of the Earth. When it is at a height $2R$ above the surface of the Earth its speed is $\frac{1}{2}\sqrt{gR}$. Assuming that air resistance can be ignored and the rocket's engine is turned off immediately after the rocket is fired,

b) find, in terms of g and R , the speed with which S was fired.

5C Simple Harmonic Motion

Simple harmonic motion (S.H.M.) is:

The equation of S.H.M.:

Speed v at displacement x , for amplitude a :

The period T:

Displacement x at time t (centre when $t = 0$ / end point when $t = 0$ / otherwise):

10. A particle is moving along a straight line with S.H.M. The amplitude of the motion is 0.8 m. It passes through the centre of the oscillation O with speed 2 m s^{-1} . Calculate:
- a) the period of the oscillation
 - b) the speed of the particle when it is 0.4 m from O
 - c) the time the particle takes to travel 0.4 m from O.

To show that a particle moves with S.H.M.:

11. A particle P of mass 0.5 kg is moving along a straight line. At time t seconds, the distance of P from a fixed point O on the line is x metres. The force acting on P has magnitude $10x$ and acts in the direction PO.
- a) Show that P is moving with simple harmonic motion.

b) Find the period of the motion.

Maximum speed and maximum acceleration:

12. A particle is moving in a straight line with simple harmonic motion. Its maximum acceleration is 12 m s^{-2} and its maximum speed is 4 m s^{-1} . Calculate:
- a) the period of the motion

b) the amplitude of the motion

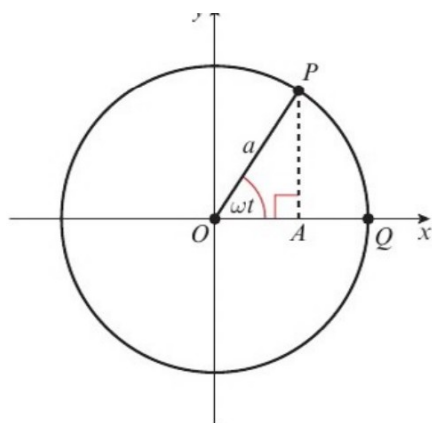
- c) the length of time, during one complete oscillation, that the particle is within $\frac{1}{3}$ m of the centre of the oscillation.

13. A small rowing boat is floating on the surface of the sea, tied to a pier. The boat moves up and down in a vertical line and it can be modelled as a particle moving with simple harmonic motion. The boat takes 2 s to travel directly from its highest point, which is 3 m below the pier, to its lowest point. The maximum speed of the boat is 3 m s^{-1} . Calculate:

- a) the amplitude of the motion

- b) the time taken by the boat to rise from its lowest point to a point 5 m below the pier.

The reference circle (geometrical method):



14. A particle P is moving with simple harmonic motion along a straight line. The centre of the oscillation is the point O, the amplitude is 0.6 m and the period is 8 s. The points A and B are points on the line of P's motion and are on opposite sides of O. The distance OA is 0.5 m and OB is 0.1 m. Calculate the time taken by P to move directly from A to B.

For an elastic string or spring on a smooth horizontal surface:

15. A particle P of mass 0.6 kg rests on a smooth horizontal floor attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 16 N. The other end of the string is fixed to a point A on the floor. The particle is pulled away from A until AP measures 1.2 m and released.

- Show that, while the string remains taut, P moves with simple harmonic motion.
- Calculate the speed of P when the string returns to its natural length.
- Calculate the time that elapses between the point where the string becomes slack and the point where it next becomes taut.
- Calculate the time taken by the particle to return to its starting point for the first time.

Two springs or strings between two fixed points:

16. A particle P of mass 0.8 kg is attached to the ends of two identical light elastic springs of natural length 1.6 m and modulus of elasticity 16 N. The free ends of the springs are attached to two points A and B which are 4 m apart on a smooth horizontal surface. The point C lies between A and B such that ABC is a straight line and $AC = 2.4$ m. The particle is held at C and then released from rest.

a) Show that the subsequent motion is simple harmonic motion.

b) Find the period and amplitude of the motion.

c) Calculate the maximum speed of P.

5E Vertical Oscillation

A particle hanging on a spring or string, pulled down and released:

In equilibrium the extension e satisfies:

Spring compared with string:

17. A particle P of mass 1.2 kg is attached to one end of a light elastic spring of modulus of elasticity 60 N and natural length 60 cm. The other end of the spring is attached to a fixed point A on a ceiling. The particle hangs in equilibrium at the point B.

a) Find the extension of the spring.

The particle is now raised vertically a distance 15 cm and released from rest.

b) Prove that P will move with simple harmonic motion.

c) Find the period and amplitude of the motion.

d) Find the speed of P as it passes through B.

- e) Find the speed of P at the instant when the spring has returned to its natural length.

Elastic potential energy:

Energy considerations:

18. A particle P of mass 0.2 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 8 N. The other end of the string is fixed to a point A on a ceiling. When the particle is hanging in equilibrium the length of the string is L m.

- a) Calculate the value of L.

The particle is held at A and released from rest. It first comes to instantaneous rest when the length of the string is K m.

- b) Use energy considerations to calculate the value of K.

- c) Show that while the string is taut, P is moving with simple harmonic motion.

The string becomes slack again for the first time T seconds after P was released from A .

d) Calculate the value of T .