

4A Acceleration Varying with Time

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Watch out when you integrate:

1. A particle P starts from rest at a point O and moves along a straight line. At time t seconds the acceleration, a m s^{-2} , of P is given by

$$a = \frac{6}{(t+2)^2}, \quad t \geq 0.$$

- a) Find the velocity of P at time t seconds.

- b) Show that the displacement of P from O when $t = 6$ is $(18 - 12 \ln 2)$ m.

2. A particle P is moving along the x-axis. At time $t = 0$, the particle is at the origin O and is moving with speed 2 m s^{-1} in the direction Ox. At time t seconds, where $t \geq 0$, the acceleration of P is $4e^{-0.5t} \text{ m s}^{-2}$ directed away from O.
- a) Find the velocity of P at time t seconds.

b) Show that the speed of P cannot exceed 10 m s^{-1} .

c) Sketch a velocity-time graph to illustrate the motion of P.



Terminal (or limiting) velocity:

3. A particle P is moving along the x-axis. At time t seconds, the velocity of P is $v \text{ m s}^{-1}$, where $v = 4 \sin(2\pi t)$. When $t = 0$, P is at O. Find:
- a) the magnitude of the acceleration of P when $t = \frac{2}{3}$
- b) the greatest distance from O attained by P during the motion.

4. A particle P is moving along the x-axis. Initially P is at the origin O. At time t seconds (where $t \geq 0$) the velocity, $v \text{ m s}^{-1}$, of P is given by $v = te^{-t/4}$. Find the distance of P from O when the acceleration of P is zero.

5. A particle is moving along the x-axis. At time t seconds the velocity of P is $v \text{ m s}^{-1}$ in the direction of x increasing, where

$$v = \begin{cases} 2t, & 0 \leq t \leq 2 \\ 2 + \frac{4}{t}, & t > 2 \end{cases}$$

When $t = 0$, P is at the origin O.

- a) Sketch a velocity-time graph to illustrate the motion of P in the interval $0 \leq t \leq 5$.



- b) Find the distance of P from O when $t = 5$.

Displacement or distance? When a particle changes direction:

6. A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(2t^2 - 7t + 3) \text{ m s}^{-1}$, $t \geq 0$. When $t = 0$, P is 10 m from the origin O. Find:

a) the values of t when P is instantaneously at rest

b) the displacement of P when $t = 5$

c) the total distance travelled by P in the interval $0 \leq t \leq 5$.



4B Acceleration Varying with Displacement

When the acceleration is a function of the displacement, $a = f(x)$, use:

Proof using the chain rule:

1. A particle P is moving on the x-axis in the direction of x increasing. When the displacement of P from the origin O is x m and its speed is v m s⁻¹, the acceleration of P is $2x$ m s⁻². When P is at O, its speed is 6 m s⁻¹. Find v in terms of x .
2. A particle P is moving along a straight line. The acceleration of P, when it has displacement x m from a fixed point O on the line and velocity v m s⁻¹, is of magnitude $4x$ m s⁻² and is directed towards O. At $x = 0$, $v = 20$. Find the values of x for which P is instantaneously at rest.

3. A particle P is moving along the positive x-axis in the direction of x increasing. When $OP = x$ m, the velocity of P is v m s⁻¹ and the acceleration of P is

$$\left(\frac{54}{x^3} - \frac{18}{x^5} \right) \text{ m s}^{-2} \quad \text{where } x \geq 1.$$

Given that $v = 6$ at $x = 1$, find v in terms of x .

4. A particle P is moving along the x-axis. Initially P is at the origin O and is moving with velocity 1 m s⁻¹ in the direction of x increasing. At time t seconds, P is x m from O, has velocity v m s⁻¹ and acceleration of magnitude $\frac{1}{2}e^{-x}$ m s⁻² directed towards O. Find:

a) v in terms of x

b) x in terms of t .

5. A particle P is moving along the positive x-axis. At $OP = x$ m, the velocity of P is v m s⁻¹ and the acceleration of P is

$$\frac{k}{(2x + 3)^2} \text{ m s}^{-2}$$

where k is a constant, directed away from O. At $x = 1$, $v = 10$ and at $x = 6$, $v = \sqrt{120}$.

- a) Find the value of k .

- b) Show that the speed of P cannot exceed $\sqrt{130}$ m s⁻¹.

4C Acceleration Varying with Velocity

When the acceleration is a function of the velocity, $a = f(v)$, and the question involves time, use:

1. A van moves along a straight horizontal road. At time t seconds, $t \geq 0$, the acceleration of the van is

$$\frac{625 - v^2}{200} \text{ m s}^{-2},$$

where $v \text{ m s}^{-1}$ is the velocity of the van. The van starts from rest.

- a) Find v in terms of t .

- b) Show that the speed of the van cannot exceed 25 m s^{-1} .

Terminal (or limiting) speed:

If you have to find distances when the acceleration varies with velocity, use:

2. A particle P moves in a straight line. When the velocity of the particle is $v \text{ m s}^{-1}$, the acceleration of the particle is given by $a = 4v \text{ m s}^{-2}$. Find the distance moved by P as the velocity increases from 10 m s^{-1} to 15 m s^{-1} .

3. A particle P moves along the positive x-axis. At time t seconds, the acceleration of the particle is $-(k^2 + v^2)$ where $v \text{ m s}^{-1}$ is the velocity of the particle and k is a positive constant. When $t = 0$, P is at O and $v = U$. The particle comes to rest at the point A. Find, in terms of k and U :
- a) the distance OA

- b) the time P takes to travel from O to A.