

Centre of mass:

The same result for particles on the y-axis:

2B Centre of Mass of a Set of Particles Arranged in a Plane

For particles arranged in a plane, consider the x- and y-coordinates separately:

1. Find the coordinates of the centre of mass of the following system of particles:

2 kg at (1, 2); 3 kg at (3, 1); 5 kg at (4, 3)

Method 1 (taking moments about each axis):

Method 2 (using position vectors):

Key result using position vectors (masses $m_1, \dots, m_n$ with position vectors $r_1, \dots, r_n$):

2. Find the coordinates of the centre of mass of the following system of particles:

4 kg at (-1, 3); 2 kg at (-2, -4); 8 kg at (4, 0); 6 kg at (1, -3)

If a question does not specify axes or coordinates:

3. A light rectangular plate ABCD has $AB = 20$ cm and $AD = 50$ cm. Particles of masses 2 kg, 3 kg, 5 kg and 5 kg are attached to the plate at the points A, B, C and D respectively. Find the distance of the centre of mass of the loaded plate from:
- a) AD
 - b) AB
4. Particles of masses 4 kg, 3 kg, 2 kg and 1 kg are placed at the points (x, y) , $(3, 2)$, $(1, -5)$ and $(6, 0)$ respectively. Given that the centre of mass of the four particles is at the point $(2.5, -2)$, find the values of x and y .
5. Three particles of masses 2 kg, 1 kg and m kg are situated at the points $(-1, 3)$, $(2, 9)$ and $(2, -1)$ respectively. Given that the centre of mass of the three particles is at the point $(1, \bar{y})$, find:
- a) the value of m
 - b) the value of $\bar{y}$.

2C Centres of Mass of Standard Uniform Plane Laminae

Lamina:

Uniform lamina:

Axes of symmetry and the centre of mass:

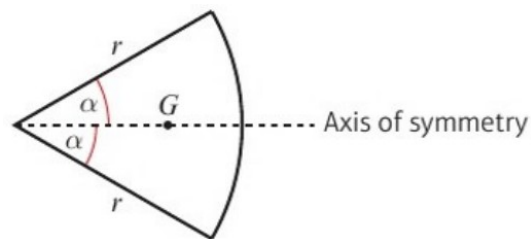
Centre of mass of a uniform circular disc:

Centre of mass of a uniform rectangular lamina:

Medians and centroid of a triangle:

Centre of mass of a uniform triangular lamina with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) :

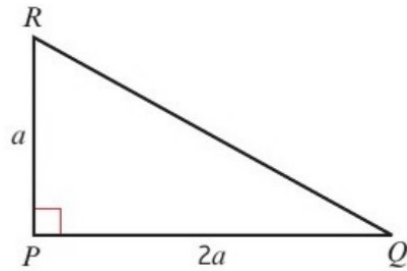
Uniform sector of a circle, radius r and centre angle 2α (α in radians):



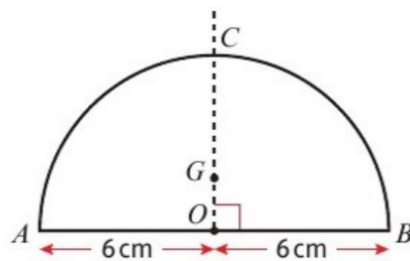
Distance of the centre of mass from the centre =

1. A uniform triangular lamina has vertices $A(1, 4)$, $B(3, 2)$ and $C(5, 3)$. Find the coordinates of its centre of mass.

2. Find the centre of mass of the uniform triangular lamina shown:



3. The diagram shows a uniform semicircular lamina of radius 6 cm with centre O . Find the centre of mass of the lamina.

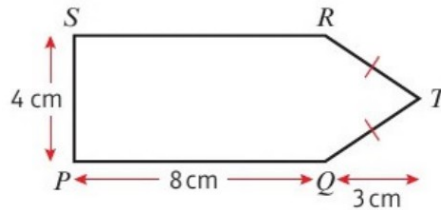


2D Centre of Mass of a Composite Lamina

Composite uniform lamina:

Method (treat each part as a particle at its own centre of mass, with mass proportional to):

1. A uniform lamina consists of a rectangle PQRS joined to an isosceles triangle QRT, as shown in the diagram. Find the distance of the centre of mass of the lamina from:

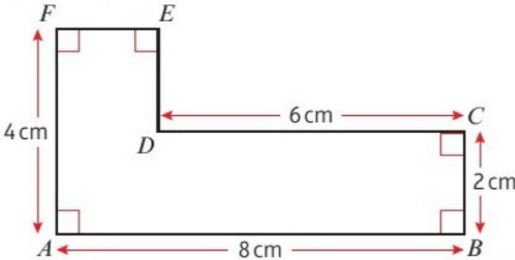


- a) PQ
- b) PS

	Rectangle PQRS	Triangle QRT	Lamina
Area			
x			
y			

The centre of mass of a uniform lamina or framework will always lie on:

2. The diagram shows a uniform lamina. Find the distance of the centre of mass of the lamina from:



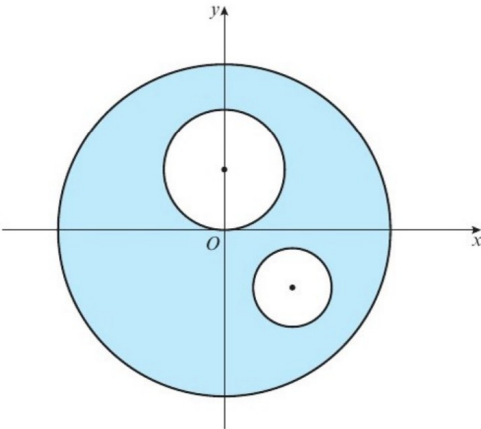
- a) AF
- b) AB

	Part 1	Part 2	Lamina
Area			
x			
y			

Two other ways of splitting this lamina:

Removing part of a lamina:

3. A uniform circular disc, centre O , of radius 5 cm has two circular holes cut in it, as shown in the diagram.



The larger hole has radius 2 cm and the smaller hole has radius 1 cm. The coordinates of the centres of the holes are $(0, 2)$ and $(2, -2)$ respectively. Find the coordinates of the centre of mass of the remaining lamina.

	Disc	Large hole	Small hole	Remaining lamina
Area				
x				
y				

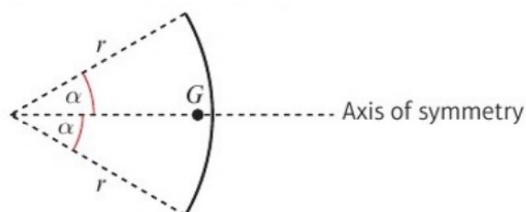
2E Centre of Mass of a Framework

Framework:

Centre of mass of a uniform straight rod:

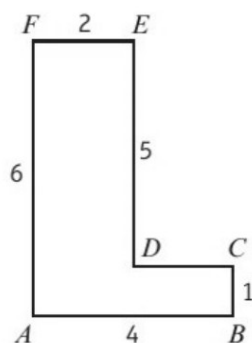
For a framework made from uniform wire, the mass of each piece is proportional to:

Uniform circular arc of radius r and centre angle 2α (α in radians):



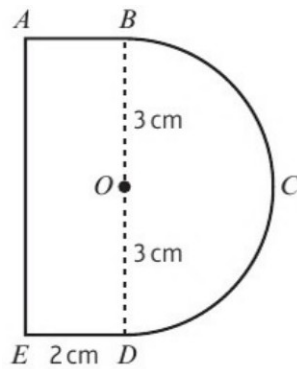
Distance of the centre of mass from the centre =

1. A framework consists of a uniform length of wire which has been bent into the shape of a letter L, as shown. Find the distance of the centre of mass of the framework from AB and AF.



	AB	BC	CD	DE	EF	FA	Frame
Length							
x							
y							

2. Find the position of the centre of mass of a framework constructed from a uniform piece of wire bent into the shape shown:



where the wire BCD is a semicircle, centre O , of radius 3 cm and wire BAED forms three sides of a rectangle ABDE.

	Arc BCD	AB	AE	ED	Framework
Length					
x					
y					

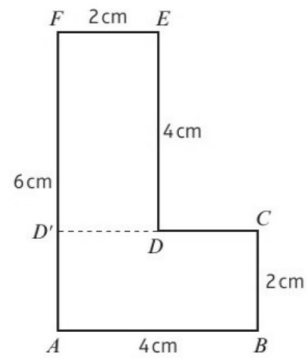
2F Lamina in Equilibrium

When a lamina or framework is freely suspended from a fixed point, or pivots freely about a horizontal axis, it rests in equilibrium with:

Why (the forces acting and their moments):

Method for finding the angle a line makes with the vertical:

1. Find the angle that the line AB makes with the vertical if this L-shaped uniform lamina is freely suspended from:



a) A

b) B

c) E

Attaching a particle to a lamina:

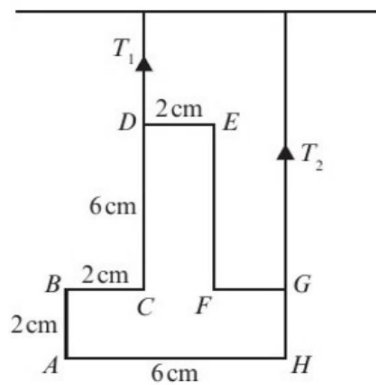
2. The L-shaped lamina in question 1 has mass M kg. Find the angle that FE makes with the vertical when a mass of $\frac{1}{10} M$ kg is attached to B and the lamina is freely suspended from F.

2G Frameworks in Equilibrium

Frameworks in equilibrium are solved:

A framework suspended by two vertical strings (resolving and taking moments):

1. The inverted T-shaped uniform framework of weight W shown in the diagram is freely suspended from D and G by two vertical strings, so that AH is horizontal. The strings and the framework lie in the same plane.



- a) Find, in terms of W , the tensions T_1 and T_2 .

The string at G breaks and the framework hangs freely in equilibrium from D .

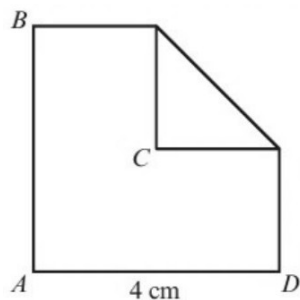
- b) Find the angle that DE makes with the vertical when the framework comes to rest in equilibrium.

2H Non-uniform Composite Laminas and Frameworks

Modelling folded card:

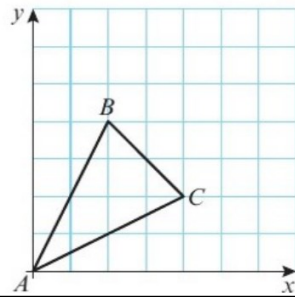
Framework made from wires of different masses:

1. The lamina shown is made from a square piece of cardboard ABCD that has had the corner C folded to the centre of the square. Find the position of the centre of mass of the lamina.



	Rectangle	Square	Triangle	Lamina
Mass				
x				
y				

2. The triangular framework shown is made from three pieces of uniform wire, AB, AC and BC of mass $2M$, $3M$ and $5M$ respectively. Find the coordinates of the centre of mass of the framework.



	AB	AC	BC	Framework
Mass				
x				
y				