

5A Motion in a Straight Line with Variable Force

When the force is a function of time, $F = f(t)$, use $F = ma$ with $a =$

When the force is a function of the displacement, $F = f(x)$, use $F = ma$ with $a =$

When the force is a function of the velocity, use $a = dv/dt$ (time involved) or $a = v dv/dx$ (distance involved). Watch out for:

1. A particle P of mass 0.5 kg is moving along the x-axis. At time t seconds the force acting on P has magnitude $(5t^2 + e^{0.2t})$ N and acts in the direction OP. When $t = 0$, P is at rest at O. Calculate:
 - a) the speed of P when $t = 2$

- b) the distance OP when $t = 3$.

2. A pebble of mass 0.2 kg is moving on a smooth horizontal sheet of ice. At time t seconds (where $t \geq 0$) a horizontal force of magnitude $2t^2 \text{ N}$ and constant direction acts on the pebble. When $t = 0$ the pebble is moving in the same direction as the force and has speed 6 m s^{-1} . When $t = T$ the pebble has speed 36 m s^{-1} . Calculate the value of T .
3. A particle P of mass 1.5 kg is moving in a straight line. The force acting on P has magnitude $(8 - 2 \cos x) \text{ N}$, where x metres is the distance OP , and acts in the direction OP . When P passes through O its speed is 4 m s^{-1} . Calculate the speed of P when $x = 2$.

Work done by a variable force $F(x)$ moving a particle from $x = a$ to $x = b$:

4. A stone S of mass 0.5 kg is moving in a straight line on a smooth horizontal floor. When S is a distance x metres from a fixed point on the line, A , a force of magnitude $(5 + 7 \cos x) \text{ N}$ acts on S in the direction AS . Given that S passes through A with speed 2 m s^{-1} , calculate:
- a) the speed of S as it passes through the point B , where $x = 3$

b) the work done by the force in moving S from A to B.

5. A particle P of mass 0.5 kg moves in a straight horizontal line. When the speed of P is $v \text{ m s}^{-1}$, the resultant force acting on P is a resistance of magnitude $3v \text{ N}$. Find the distance moved by P as it slows down from 12 m s^{-1} to 6 m s^{-1} .

6. A particle P of mass m is moving along Ox in the direction of x increasing. At time t , the only force acting on P is a resistance of magnitude $mk(c^2 + v^2)$, where v is the speed of P and c and k are positive constants. When $t = 0$, P is at O and $v = U$. The particle P comes to rest at the point A. Find:
- a) the distance OA

b) the time P takes to travel from O to A.

7. A car of mass 800 kg travels along a straight horizontal road. The engine of the car produces a constant driving force of magnitude 2000 N. At time t seconds, the speed of the car is $v \text{ m s}^{-1}$. As the car moves, the total resistance to the motion of the car is of magnitude $(400 + 4v^2) \text{ N}$. The car starts from rest.

a) Find v in terms of t .

b) Show that the speed of the car cannot exceed 20 m s^{-1} .