

4B Acceleration Varying with Displacement

When the acceleration is a function of the displacement, $a = f(x)$, use:

Proof using the chain rule:

1. A particle P is moving on the x-axis in the direction of x increasing. When the displacement of P from the origin O is x m and its speed is v m s⁻¹, the acceleration of P is $2x$ m s⁻². When P is at O, its speed is 6 m s⁻¹. Find v in terms of x .
2. A particle P is moving along a straight line. The acceleration of P, when it has displacement x m from a fixed point O on the line and velocity v m s⁻¹, is of magnitude $4x$ m s⁻² and is directed towards O. At $x = 0$, $v = 20$. Find the values of x for which P is instantaneously at rest.

3. A particle P is moving along the positive x-axis in the direction of x increasing. When $OP = x$ m, the velocity of P is v m s⁻¹ and the acceleration of P is

$$\left(\frac{54}{x^3} - \frac{18}{x^5} \right) \text{ m s}^{-2} \quad \text{where } x \geq 1.$$

Given that $v = 6$ at $x = 1$, find v in terms of x .

4. A particle P is moving along the x-axis. Initially P is at the origin O and is moving with velocity 1 m s⁻¹ in the direction of x increasing. At time t seconds, P is x m from O, has velocity v m s⁻¹ and acceleration of magnitude $\frac{1}{2}e^{-x}$ m s⁻² directed towards O. Find:

a) v in terms of x

b) x in terms of t .

5. A particle P is moving along the positive x-axis. At $OP = x$ m, the velocity of P is v m s⁻¹ and the acceleration of P is

$$\frac{k}{(2x + 3)^2} \text{ m s}^{-2}$$

where k is a constant, directed away from O. At $x = 1$, $v = 10$ and at $x = 6$, $v = \sqrt{120}$.

a) Find the value of k .

b) Show that the speed of P cannot exceed $\sqrt{130}$ m s⁻¹.