

3A Using Calculus to Find Centres of Mass

For a system of particles $m_1, m_2, \dots$ at $(x_1, y_1), (x_2, y_2), \dots$:

Elemental strip:

ρ (rho) can mean:

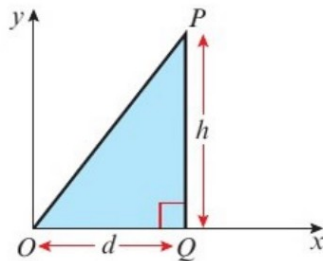
Centre of mass of a uniform lamina bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$:

$$\bar{x} =$$

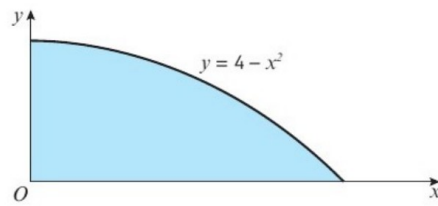
$$\bar{y} =$$

Why is the strip's centre of mass at height $\frac{1}{2}y$?

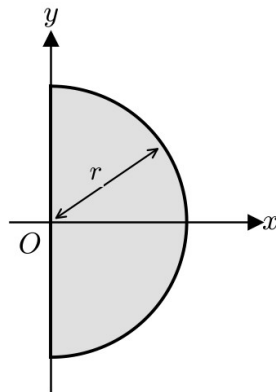
1. Use calculus to find the position of the centre of mass of a right-angled triangular lamina OPQ with base d and height h , as shown in the diagram.



2. Find the coordinates of the centre of mass of the uniform lamina bounded by the curve with equation $y = 4 - x^2$, the x-axis and the y-axis, as shown.

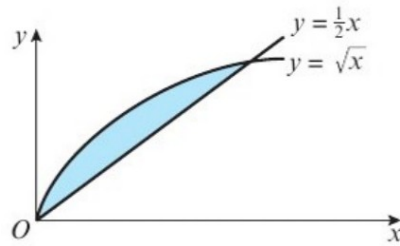


3. A uniform semicircular lamina has radius r cm. Find the position of its centre of mass.



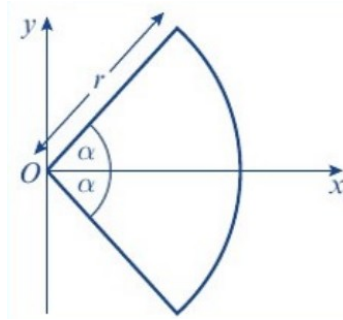
Centre of mass of a uniform semicircular lamina:

4. The diagram shows a uniform lamina occupying the shaded region bounded by the curve with equation $y = \sqrt{x}$ and the straight line with equation $y = \frac{1}{2}x$. Find the coordinates of the centre of mass of the lamina.

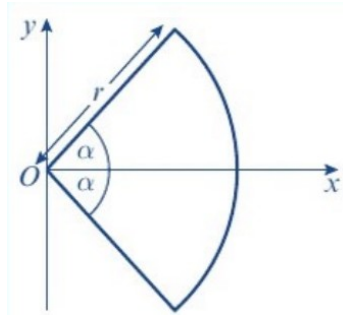


Lamina between two curves y_1 (upper) and y_2 (lower): mass of strip and centre of mass of strip

5. Find the centre of mass of a uniform lamina in the form of a sector of a circle, radius r and centre O , which subtends an angle 2α at O .



6. Find the centre of mass of a uniform wire in the form of an arc of a circle, radius r and centre O , which subtends an angle 2α at O .



Standard results (in the formula booklet), distance from O along the axis of symmetry:

	Distance of centre of mass from O
Sector of a circle, angle 2α	
Arc of a circle, angle 2α	
Semicircular lamina	