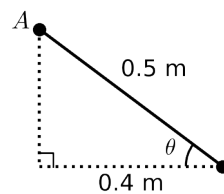


1C Three-Dimensional Problems with Objects Moving in Horizontal Circles

Method: which directions to resolve in, and what each equation says:

Conical pendulum:

1. A particle of mass 2 kg is attached to one end of a light inextensible string of length 50 cm. The other end of the string is attached to a fixed point A. The particle moves with constant angular speed in a horizontal circle of radius 40 cm. The centre of the circle is vertically below A. Calculate the tension in the string and the angular speed of the particle.

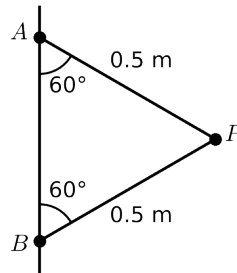


2. A particle of mass m is attached to one end of a light inextensible string of length l . The other end of the string is attached to a fixed point A. The particle moves with constant angular speed in a horizontal circle. The string is taut and the angle between the string and the vertical is θ . The centre of the circle is vertically below A. Find the angular speed of the particle.

Banked tracks: if there is no friction (no tendency to slip), the force towards the centre comes from:

3. A car travels round a bend of radius 500 m on a road which is banked at an angle θ to the horizontal. The car is assumed to be moving at constant speed in a horizontal circle and there is no tendency to slip. If there is no frictional force acting on the car when it is travelling at 90 km h^{-1} , find the value of θ .

4. The diagram shows a particle P of mass m attached by two strings to fixed points A and B, where A is vertically above B. The strings are both taut and P is moving in a horizontal circle with constant angular speed $2\sqrt{3g}$ rad s^{-1} . Both strings are 0.5 m in length and inclined at 60° to the vertical.



- a) Calculate the tensions in the two strings.

The strings will break if the tension in them exceeds $8mg$ N. The angular speed of the particle is increased until the strings break.

- b) State which string will break first.

- c) Find the maximum angular speed of the particle before the string breaks.

A banked aircraft: the lift force acts perpendicular to the wings, so its components do two jobs:

5. An aircraft of mass 2 tonnes flies at 500 km h^{-1} on a path which follows a horizontal circular arc in order to change course from due north to due east. The aircraft turns in the clockwise direction from due north to due east. It takes 40 seconds to change course, with the aircraft banked at an angle α to the horizontal. Calculate the value of α and the magnitude of the lift force perpendicular to the surface of the aircraft's wings.

Rough surfaces: finding the greatest (or least) possible speed, and which way friction acts:

6. In this question use $g = 9.81 \text{ m s}^{-2}$. A hollow right circular cone is fixed with its axis of symmetry vertical and its vertex V pointing downwards. A particle P of mass 25 g moves in a horizontal circle with centre C , and radius 0.27 m , on the rough inner surface of the cone. P remains in contact as it moves with constant angular speed, ω , and does not slip. The angle between VP and the vertical is θ , such that $\tan \theta = 0.45$. The coefficient of friction between the particle and the cone is 0.15 . Find the greatest possible value of ω .

