

6A Play-Safe Strategies and Stable Solutions

Two-person game:

Pay-off matrix:

The Prisoner's Dilemma (two prisoners each choose to confess or not to confess):

Playing safe: each player looks for ...

Pure strategy:

1. In this pay-off matrix player A has a choice of four options and player B has a choice of three options. The outcomes are given as ordered pairs, (A's winnings, B's winnings). Determine the play-safe strategy for each player.

	B plays 1	B plays 2	B plays 3	Worst outcome for A
A plays 1	(8, 2)	(0, 9)	(7, 3)	
A plays 2	(3, 6)	(9, 0)	(2, 7)	
A plays 3	(1, 7)	(6, 4)	(8, 1)	
A plays 4	(4, 2)	(4, 6)	(5, 1)	
Worst outcome for B				

Collaborating / knowing the other player's strategy:

Zero-sum game:

A pay-off matrix is always written from ...

The play-safe strategies are:

- For player A, always play the row containing the ...
- For player B, always play the column containing the ...

Row maximin:

Column minimax:

2. Determine the play-safe strategy for both players, A and B, for the game below.

	B plays 1	B plays 2	B plays 3	Row minimum
A plays 1	3	-4	2	
A plays 2	-1	4	-2	
A plays 3	-3	1	4	
A plays 4	1	-1	1	
Column maximum				

Stable solution:

Equilibrium:

3. Anna and Ben play a two-person zero-sum game, which is represented by the following pay-off matrix. Find the play-safe strategy for each player and show that the game has a stable solution.

	B plays 1	B plays 2	B plays 3	B plays 4	Row minimum
A plays 1	4	-1	2	3	
A plays 2	4	6	3	7	
A plays 3	1	2	-2	4	
Column maximum					

Saddle point:

Stable solution theorem: in a two-person zero-sum game there will be a stable solution if and only if

If a two-person zero-sum game has a stable solution:

- it must occur when ...
- the value of the game to player A is ...
- the value of the game to player B is ...

4. A two-player zero-sum game has the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3	Row minimum
A plays 1	7	-4	-2	
A plays 2	-3	2	0	
A plays 3	5	5	6	
Column maximum				

a) Show that there is a stable solution.

b) Find the value of the game to A.

5. A two-player zero-sum game has the following pay-off matrix:

	B plays 1	B plays 2	Row minimum
A plays 1	4	-2	
A plays 2	-5	3	
Column maximum			

a) Determine the play-safe strategy for each player.

b) Verify that there is no stable solution for this game.

c) State the pay-off for each player if they both play safe.

d) Determine the pay-off matrix for B.

	A plays 1	A plays 2
B plays 1		
B plays 2		

To write the pay-off matrix for player B:

6B Reducing the Pay-off Matrix

Row dominance: if every entry in row p is ... than the corresponding entry in row q , then

Column dominance: if every entry in column r is ... than the corresponding entry in column s , then

Why the column with the smaller values dominates:

Reducing the pay-off matrix using dominance arguments:

1. A and B play a zero-sum game, represented by the pay-off matrix below. Explain why the 3×2 game can be reduced to a 2×2 game.

	B plays 1	B plays 2
A plays 1	7	1
A plays 2	-3	6
A plays 3	4	0

Watch out: you can only use dominance arguments to delete a row (or column) if ...

2. A and B play a zero-sum game, represented by the pay-off matrix below. Explain why the 2×3 game can be reduced to a 2×2 game.

	B plays 1	B plays 2	B plays 3
A plays 1	7	1	-2
A plays 2	-3	6	1

Watch out: when you delete a column, the headings ...

6C Optimal Strategies for Games with No Stable Solution

Mixed strategy:

Expected pay-off:

1. A two-player zero-sum game is represented by the following pay-off matrix.

	B plays 1	B plays 2
A plays 1	4	-2
A plays 2	-5	3

Player A is considering three different mixed strategies:

Strategy X: play row 1 with probability 0.5 and row 2 with probability 0.5

Strategy Y: play row 1 with probability 0.6 and row 2 with probability 0.4

Strategy Z: play row 1 with probability 0.9 and row 2 with probability 0.1

Find the expected pay-offs for each mixed strategy.

	If B plays 1	If B plays 2
X		
Y		
Z		

Optimal mixed strategy for player A:

Value of a game with no stable solution:

To find the optimal mixed strategy for a player with two choices in a $2 \times n$ or $n \times 2$ game:

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-
-
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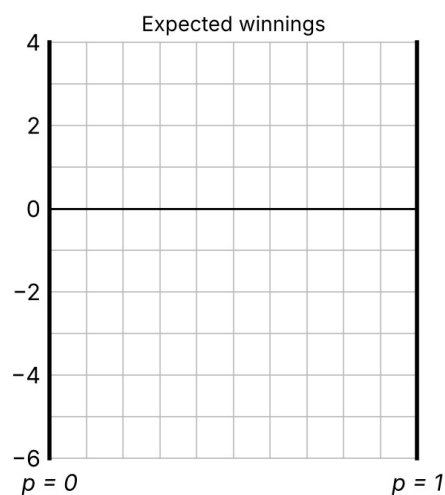
Rugby posts:

2. A two-person zero-sum game is represented by the following pay-off matrix:

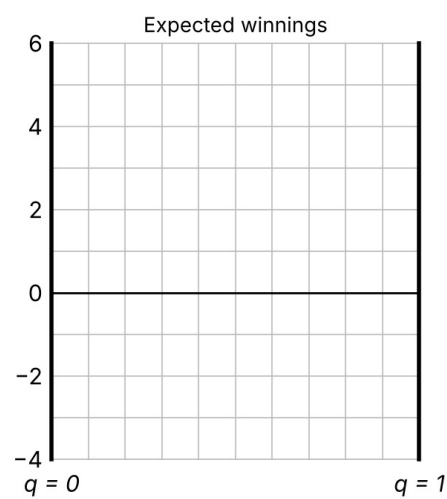
	B plays 1	B plays 2
A plays 1	4	-2
A plays 2	-5	3

Find the optimal mixed strategy a for player A, b for player B. In each case state the value of the game to that player.

a) For player A



b) For player B



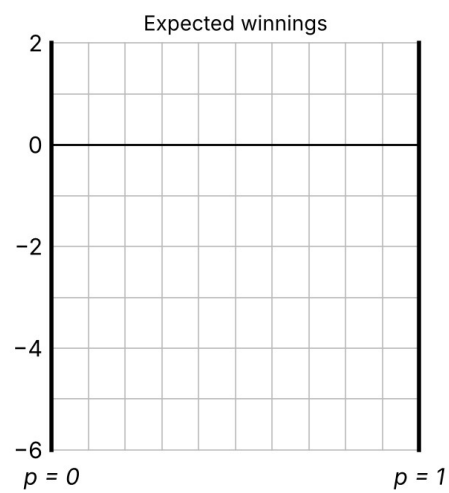
In a two-person zero-sum game, if the value of the game to player A is v , then the value of the game to player B will be

3. Alf and Bert play a zero-sum game, represented by the pay-off matrix below.

	Bert plays 1	Bert plays 2	Bert plays 3	Row minimum
Alf plays 1	-5	2	-1	
Alf plays 2	2	-3	1	
Column maximum				

a) Verify that this game has no stable solution.

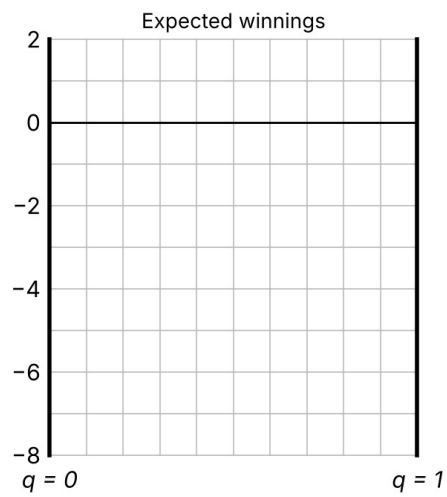
b) Determine Alf's best strategy and the value of the game to him.



If there is more than one intersection point, the best strategy is given by ...

4. A and B play a zero-sum game, given by the pay-off matrix below. Determine B's best strategy, and the value of the game to B.

	B plays 1	B plays 2	Row minimum
A plays 1	4	-1	
A plays 2	3	4	
A plays 3	1	7	
A plays 4	-2	8	
Column maximum			



Watch out: the optimal solution is not necessarily ...

6D Converting Games to Linear Programming Problems

When do you formulate a game as a linear programming problem?

Before formulating, always check for ...

To formulate a game as a linear programming problem:

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1. Alf and Bert play a zero-sum game, represented by the pay-off matrix below.

	B plays 1	B plays 2	B plays 3	Row minimum
A plays 1	-2	4	2	
A plays 2	0	-3	-2	
A plays 3	-6	1	3	
Column maximum				

- a) Formulate the game above as a linear programming problem for player A, writing the constraints as equations and defining your variables clearly.

	B plays 1	B plays 2	B plays 3
A plays 1			
A plays 2			
A plays 3			

b) Write down an initial simplex tableau for this problem.

	V	p₁	p₂	p₃	r	s	t	u	Value
r									
s									
t									
u									
p									

c) Outline how you would change the method to formulate the game as a linear programming problem for player B. (You do not need to formulate the problem.)

	A plays 1	A plays 2	A plays 3
B plays 1			
B plays 2			
B plays 3			

Why replace $p_1 + p_2 + p_3 = 1$ by $p_1 + p_2 + p_3 \leq 1$?

2. Jenny and Merry play a zero-sum game, represented by the pay-off matrix for Jenny shown below.

	Merry plays 1	Merry plays 2	Merry plays 3	Row minimum
Jenny plays 1	-3	0	2	
Jenny plays 2	-2	2	1	
Jenny plays 3	0	-3	-2	
Column maximum				

- a) Convert the game into a linear programming problem for Jenny, defining your decision variables. You should write the constraints as equations.

	M plays 1	M plays 2	M plays 3
J plays 1			
J plays 2			
J plays 3			

- b) Write down an initial simplex tableau, making your variables clear.

	V	p_1	p_2	p_3	r	s	t	u	Value
r									
s									
t									
u									
P									

- c) Using the simplex algorithm, the solution obtained is $p_1 = 0$, $p_2 = 3/7$, $p_3 = 4/7$. Find the value of the game to Jenny.

Watch out: at the end you must ...