

2A The Hungarian Algorithm

Allocation problem:

Cost matrix:

To reduce the cost matrix:

Optimal allocation:

1. The table shows the times taken, in minutes, by four workers to complete each task of a four-stage production process.

	Task W	Task X	Task Y	Task Z
Kris	12	23	15	40
Laura	14	21	17	20
Sam	13	22	20	30
Steve	14	24	13	10

The time taken for the process is the sum of the times of the four separate tasks. Reduce the cost matrix.

Reduce rows:

	Task W	Task X	Task Y	Task Z
Kris				
Laura				
Sam				
Steve				

Reduce columns:

	Task W	Task X	Task Y	Task Z
Kris				
Laura				
Sam				
Steve				

Optimal allocation and total time:

The Hungarian algorithm:

2. Barbara, Percival and Spike do garden maintenance. The table shows the times, in minutes, that they would take to do each task in Mrs Green's garden. Allocate the tasks so that the time taken to do the whole job is as small as possible.

	Dig vegetable patch	Weed flower beds	Cut lawn and hedges
Barbara	250	80	160
Percival	230	90	150
Spike	230	110	140

	Dig vegetable patch	Weed flower beds	Cut lawn and hedges
Barbara			
Percival			
Spike			

	Dig vegetable patch	Weed flower beds	Cut lawn and hedges
Barbara			
Percival			
Spike			

	Dig vegetable patch	Weed flower beds	Cut lawn and hedges
Barbara			
Percival			
Spike			

The short cut (augmenting the reduced cost matrix by e):

3. Five workers Ben, Ellie, Greg, Hyo and Toby are to be assigned to five tasks involved in making a soft toy. The table shows the times in minutes taken to complete each task.

	Cut	Sew	Fill	Finish	Pack
Ben	9	8	3	6	10
Ellie	5	5	7	5	5
Greg	10	9	3	9	10
Hyo	10	7	2	9	7
Toby	9	8	2	7	10

Reducing rows first, use the Hungarian algorithm to determine an allocation that minimises the total time.

	Cut	Sew	Fill	Finish	Pack
Ben					
Ellie					
Greg					
Hyo					
Toby					

	Cut	Sew	Fill	Finish	Pack
Ben					
Ellie					
Greg					
Hyo					
Toby					

	Cut	Sew	Fill	Finish	Pack
Ben					
Ellie					
Greg					
Hyo					
Toby					

	Cut	Sew	Fill	Finish	Pack
Ben					
Ellie					
Greg					
Hyo					
Toby					

2B Using a Dummy

If an allocation problem is not $n \times n$:

Workers allocated to dummy tasks / tasks allocated to dummy workers:

1. The table shows the time, in minutes, taken for four workers Mark, Nicky, Nigel and Susie to do each of three tasks A, B and C.

	Task A	Task B	Task C
Mark	12	23	15
Nicky	14	21	17
Nigel	13	22	20
Susie	14	24	13

Use the Hungarian algorithm to obtain an allocation that minimises the total time.

	Task A	Task B	Task C	Task D
Mark				
Nicky				
Nigel				
Susie				

	Task A	Task B	Task C	Task D
Mark				
Nicky				
Nigel				
Susie				

2C Maximum Profit Allocation

To find an allocation that maximises the total:

1. The table shows the profits, in pounds, so that the profit made when Cherry does task W is £12.

	Task W	Task X	Task Y	Task Z
Cherry	12	23	15	40
Jimmy	14	21	17	20
Mac	13	22	20	30
Plum	14	24	13	10

Find the allocation of tasks to people to maximise the total income.

	Task W	Task X	Task Y	Task Z
Cherry				
Jimmy				
Mac				
Plum				

	Task W	Task X	Task Y	Task Z
Cherry				
Jimmy				
Mac				
Plum				

	Task W	Task X	Task Y	Task Z
Cherry				
Jimmy				
Mac				
Plum				

	Task W	Task X	Task Y	Task Z
Cherry				
Jimmy				
Mac				
Plum				

A useful tip (choosing the pattern of lines):

2D Managing Incomplete Data

If it is not possible to assign a task to a given person:

1. An agency needs to assign four relief chefs Denis, Hilary, Robert and Trudy to four restaurants serving Chinese, French, Indian and Italian food. The travelling expenses, in pounds, that will be paid to each chef are shown in the table. Some of the chefs cannot work in some of the restaurants, since they are not familiar with that style of cookery.

	New Dell (Chinese)	Aye Full Tower (French)	Hows a Curry (Indian)	Piece a Pasta (Italian)
Denis	–	27	15	40
Hilary	14	21	17	13
Robert	20	–	13	–
Trudy	14	24	10	30

Use the Hungarian algorithm, reducing rows first, to obtain an allocation that minimises the total travelling expenses paid.

	New Dell (Chinese)	Aye Full Tower (French)	Hows a Curry (Indian)	Piece a Pasta (Italian)
Denis				
Hilary				
Robert				
Trudy				

	New Dell (Chinese)	Aye Full Tower (French)	Hows a Curry (Indian)	Piece a Pasta (Italian)
Denis				
Hilary				
Robert				
Trudy				

	New Dell (Chinese)	Aye Full Tower (French)	Hows a Curry (Indian)	Piece a Pasta (Italian)
Denis				
Hilary				
Robert				
Trudy				

	New Dell (Chinese)	Aye Full Tower (French)	Hows a Curry (Indian)	Piece a Pasta (Italian)
Denis				
Hilary				
Robert				
Trudy				

	New Dell (Chinese)	Aye Full Tower (French)	Hows a Curry (Indian)	Piece a Pasta (Italian)
Denis				
Hilary				
Robert				
Trudy				

Incomplete data when maximising:

2. A team of four workers A, B, C and D are to be assigned to four tasks 1, 2, 3 and 4. Each worker can only be assigned to one task and each task is to be completed by one worker. Worker B cannot be assigned to task 3. The table shows the profit, in pounds, to be made by each worker on each task.

	1	2	3	4
A	47	68	125	52
B	39	72	-	60
C	48	63	140	63
D	37	75	132	68

- a) Reducing rows first, use the Hungarian algorithm to allocate workers to tasks so that the profit is maximised. Show the table at each stage.

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

b) Work out the maximum profit.

2E Linear Programming

Decision variables (binary coding):

Presenting an allocation problem as a linear programming problem:

1. Four workers Amy, Bob, Chris and Dave are to be assigned to tasks 1, 2, 3 and 4. Each worker is assigned to just one task and each task is to be completed by one worker. The time, in minutes, taken by each worker to complete each task is shown in the table. The total time taken is to be minimised.

	Task 1	Task 2	Task 3	Task 4
Amy	14	17	21	25
Bob	18	15	19	22
Chris	16	14	17	28
Dave	20	18	20	24

Formulate this as a linear programming problem.

2. The table shows the time, in minutes, taken for four workers A, B, C and D to do each of three tasks P, Q and R. Each worker can only be assigned one task, and each task is to be completed by one worker. Worker B cannot be assigned to task R and worker D cannot be assigned to task Q.

	P	Q	R
A	10	14	20
B	12	14	-
C	11	16	22
D	13	-	21

Formulate the problem as a linear programming problem, defining your variables and making the objective and constraints clear.

Maximisation problems:

3. Four machines 1, 2, 3 and 4 are to be used to perform four tasks A, B, C and D. Each machine is to be assigned to just one task and each task must be assigned to just one machine. The profit generated when each machine performs each task is given in the table. The company wishes to maximise the total profit.

	A	B	C	D
1	10	2	8	6
2	9	3	11	3
3	3	1	4	2
4	3	2	1	5

Formulate the above assignment problem as a linear programming problem, defining your variables and making the objective and constraints clear.

