

1A The North-West Corner Method

In a transportation problem, what is meant by:

the supply (or stock):

the demand:

the unit cost:

1. Three suppliers A, B and C each produce road grit which has to be delivered to council depots W, X, Y and Z. The stock held at each supplier and the demand from each depot is known. The cost, in pounds, of transporting one lorry load of grit from each supplier to each depot is also known. This information is given in the table.

	W	X	Y	Z	Stock
A	180	110	130	290	14
B	190	250	150	280	16
C	240	270	190	120	20
Demand	11	15	14	10	50

Use the information in the table to write down:

- a) the number of lorry loads of grit that each supplier can supply
- b) the number of lorry loads of grit required at each depot
- c) the cost of transporting a lorry load of grit from A to W
- d) the cost of transporting a lorry load of grit from C to Z
- e) Which is the cheapest route to use?
- f) Which is the most expensive route to use?

The transportation algorithm (the five steps):

The north-west corner method:

Number of occupied cells (routes used) for m supply points and n demand points:

2. Use the north-west corner method to find an initial solution to the problem described in question 1, and state its cost.

Units transported

	W	X	Y	Z	Stock
A					14
B					16
C					20
Demand	11	15	14	10	50

1B Unbalanced Problems and Degenerate Solutions

Unbalanced transportation problem:

If total supply > total demand:

If total supply < total demand:

The demand (or supply) and the costs for a dummy:

What units sent to a dummy demand point, or from a dummy supply point, represent:

1. Three outlets A, B and C are supplied by three suppliers X, Y and Z. The table shows the cost, in pounds, of transporting each unit, the number of units required at each outlet and the number of units available at each supplier.

	A	B	C	Supply
X	9	11	10	40
Y	10	8	12	60
Z	12	7	8	50
Demand	50	40	30	

a) Explain why it is necessary to add a dummy demand point in order to solve this problem.

b) Add a dummy demand point and appropriate costs to the table.

	A	B	C		Supply
X	9	11	10		40
Y	10	8	12		60
Z	12	7	8		50
Demand	50	40	30		

c) Use the north-west corner method to obtain an initial solution.

	A	B	C		Supply
X					40
Y					60
Z					50
Demand	50	40	30		

2. The table shows the cost in pounds of transporting each unit to three retailers A, B and C from three suppliers W, X and Y. It also shows the number of units required by each retailer and the number available at each supplier.

	A	B	C	Supply
W	8	9	11	55
X	9	10	12	70
Y	11	10	8	65
Demand	74	72	68	

- a) Use the north-west corner method to obtain an initial solution.

	A	B	C	Supply
W	8	9	11	55
X	9	10	12	70
Y	11	10	8	65
Demand	74	72	68	

Units transported

	A	B	C	Supply
W				55
X				70
Y				65
Demand	74	72	68	

- b) State, with a reason, which retailer has not had their demand fully met.

Degenerate solution (m rows and n columns):

When this happens:

How to make the solution non-degenerate:

3. The table shows the unit cost in pounds of transporting goods from supply points W, X, Y and Z to demand points A, B and C. It also shows the number of units required at each demand point and the number of units available at each supply point.

	A	B	C	Supply
W	10	11	6	30
X	4	5	9	20
Y	3	8	7	35
Z	11	10	9	35
Demand	30	40	50	120

- a) Demonstrate that the north-west corner method gives a degenerate solution and explain why it is degenerate.

	A	B	C	Supply
W				30
X				20
Y				35
Z				35
Demand	30	40	50	120

- b) Adapt your solution to give a non-degenerate initial solution and state its cost.

	A	B	C	Supply
W				30
X				20
Y				35
Z				35
Demand	30	40	50	120

1C Finding an Improved Solution

To find an improved solution you need to:

Shadow costs:

For each used route from source P to destination Q:

Finding the shadow costs (steps):

1. Calculate the shadow costs given by the initial solution of the problem in Example 2 (1A question 2). The costs and the initial solution are shown below.

	W	X	Y	Z	Stock
A	180	110	130	290	14
B	190	250	150	280	16
C	240	270	190	120	20
Demand	11	15	14	10	50

	W	X	Y	Z	Stock
A	11	3			14
B		12	4		16
C			10	10	20
Demand	11	15	14	10	50

Shadow costs (write the costs of the used routes in the grid first)

	W	X	Y	Z	S(·)
A					
B					
C					
D(·)					

2. Calculate the shadow costs given by the initial solution of the problem in Example 3 (1B question 1), shown in the tables below.

	A	B	C	D	Supply
X	9	11	10	0	40
Y	10	8	12	0	60
Z	12	7	8	0	50
Demand	50	40	30	30	150

	A	B	C	D	Supply
X	40				40
Y	10	40	10		60
Z			20	30	50
Demand	50	40	30	30	150

	A	B	C	D	S(·)
X					
Y					
Z					
D(·)					

Improvement index for the route PQ:

Entering cell:

If two cells tie:

When is a solution optimal?

3. Use the shadow costs found in question 1, shown in the table below, to calculate improvement indices, and use these to identify the entering cell.

	W	X	Y	Z	S(·)
A	180	110	130	290	0
B	190	250	150	280	140
C	240	270	190	120	180
D(·)	180	110	10	-60	

Improvement indices

	W	X	Y	Z	S(·)
A					0
B					140
C					180
D(·)	180	110	10	-60	

4. The table shows a transportation problem.

	X	Y	Z	Supply
A	11	12	17	11
B	13	10	13	15
C	15	18	9	14
Demand	10	15	15	40

- a) Use the north-west corner method to find an initial solution to the transportation problem shown in the table.

	X	Y	Z	Supply
A				11
B				15
C				14
Demand	10	15	15	40

b) Find the shadow costs and improvement indices.

	X	Y	Z	S(.)
A				
B				
C				
D(.)				

	X	Y	Z	S(.)
A				
B				
C				
D(.)				

c) Hence determine whether the solution is optimal.

1D The Stepping-Stone Method

The stepping-stone method (the two rules for the cycle of adjustments):

The value of θ :

The exiting cell:

If two cells are reduced to 0:

Checking the new cost:

1. In Example 2 (1A question 2) the table of costs and the initial solution were as shown, at a cost of £9010. Obtain an improved solution and find the improved cost.

	W	X	Y	Z	Stock
A	180	110	130	290	14
B	190	250	150	280	16
C	240	270	190	120	20
Demand	11	15	14	10	50

	W	X	Y	Z	Stock
A	11	3			14
B		12	4		16
C			10	10	20
Demand	11	15	14	10	50

Stepping-stone route

	W	X	Y	Z	Stock
A	11	3			14
B		12	4		16
C			10	10	20
Demand	11	15	14	10	50

Improved solution

	W	X	Y	Z	Stock
A					14
B					16
C					20
Demand	11	15	14	10	50

To find an optimal solution:

- Find an optimal solution for question 1.

Second iteration: shadow costs and improvement indices

	W	X	Y	Z	S(·)
A					
B					
C					
D(·)					

	W	X	Y	Z	S(·)
A					
B					
C					
D(·)					

Stepping-stone route and improved solution

	W	X	Y	Z	Stock
A		14			14
B	11	1	4		16
C			10	10	20
Demand	11	15	14	10	50

	W	X	Y	Z	Stock
A					14
B					16
C					20
Demand	11	15	14	10	50

Third iteration: shadow costs and improvement indices

	W	X	Y	Z	S(·)
A					
B					
C					
D(·)					

	W	X	Y	Z	S(·)
A					
B					
C					
D(·)					

3. The table shows the unit cost, in pounds, of transporting goods from each of three warehouses A, B and C to each of three supermarkets X, Y and Z. It also shows the stock at each warehouse and the demand at each supermarket. Solve the transportation problem shown in the table. Use the north-west corner method to obtain an initial solution. You must state your shadow costs, improvement indices, stepping-stone routes, θ values, entering cells and exiting cells. You must state the initial cost and the improved cost after each iteration.

	X	Y	Z	Stock
A	24	22	28	13
B	26	26	14	11
C	20	22	20	12
Demand	10	13	13	36

Initial solution

	X	Y	Z	Stock
A				13
B				11
C				12
Demand	10	13	13	36

First iteration

	X	Y	Z	S(·)
A				
B				
C				
D(·)				

	X	Y	Z	S(·)
A				
B				
C				
D(·)				

	X	Y	Z	Stock
A				13
B				11
C				12
Demand	10	13	13	36

	X	Y	Z	Stock
A				13
B				11
C				12
Demand	10	13	13	36

Second iteration

	X	Y	Z	S(·)
A				
B				
C				
D(·)				

	X	Y	Z	S(·)
A				
B				
C				
D(·)				

	X	Y	Z	Stock
A				13
B				11
C				12
Demand	10	13	13	36

	X	Y	Z	Stock
A				13
B				11
C				12
Demand	10	13	13	36

Third iteration

	X	Y	Z	S(·)
A				
B				
C				
D(·)				

	X	Y	Z	S(·)
A				
B				
C				
D(·)				

1E Linear Programming

Decision variables for a transportation problem:

The objective:

Supply constraints use:

Demand constraints use:

The standard way of presenting a transportation problem as a linear programming problem:

Balanced problems and equalities:

Sigma notation:

1. Formulate the transportation problem below as a linear programming problem. You must state your decision variables, objective and constraints.

	R	S	T	Supply
A	3	3	2	25
B	4	2	3	40
C	3	4	3	31
Demand	30	30	36	96